

Computational nonlinear thermoelastic deformation and undamped natural frequency analysis of a tensile nylon V-actuator

J. Murín^{a,*}, V. Goga^a, J. Hrabovský^a, J. Paulech^a, G. Gálik^a, M. Uličný^a,
V. Kutíš^a, S. Kugler^b

^a*Institute of Automotive Mechatronics, Faculty of Electrical Engineering and Information Technology,
Slovak University of Technology in Bratislava, Ilkovičova 3, 812 19 Bratislava, Slovak Republic*

^b*University of Applied Sciences, Wiener Neustadt, Johannes Gutenberg Strasse 3, Wiener Neustadt A/2700, Austria*

Received 11 February 2026; accepted 8 October 2026

Abstract

This article investigates the effects of geometric nonlinearity and thermal loading on the thermal stroke and natural vibration of a tensile nylon actuator. The proposed single-degree-of-freedom configuration consists of two identical tensile nylon springs arranged in a V-shape (V-actuator). Loaded by an attached gravitational mass at its actuation point and subjected to thermal loading, the actuator exhibits both negative thermal expansion and compressive prestress. The thermomechanical actuating force induced by the thermal shortening of the actuator springs lifts the attached external mass. This setup demonstrates a significant increase in both the thermal stroke and tensile action force compared to a parallel spring arrangement under an identical gravitational load. The natural frequency of the actuator's undamped small vibrations is determined from the static deflection induced by the external gravitational mass and the thermal state of the springs. A computational model is developed based on the measured tensile characteristics of the individual springs and the complete V-actuator assembly. Utilising the updated Lagrangian formulation (ULF) for large displacements and a custom nylon twisted coiled spring (NTCS) finite element, nonlinear incremental equations are established to calculate the elastic extension, thermoelastic shortening, and total static deflection of the actuator's operating point. These equations are solved using the incremental Newton method. The results of this nonlinear algorithm are compared with a classical linear spring model and selected numerical simulations are validated through experimental measurements on a physical prototype. The findings reveal that geometric nonlinearity and heating significantly influence the actuator's thermal stroke and the natural frequency of the oscillating attached mass.

© 2026 University of West Bohemia in Pilsen.

Keywords: nylon twisted coiled spring, artificial muscle, tensile thermoelastic nylon V-actuator, negative thermal expansion, geometric nonlinearity, thermal stroke, natural undamped vibration

1. Introduction

Tensile coiled springs wound from nylon fibre (fishing line) are characterized by negative thermal expansion and significant passive damping [4, 8, 9]. Due to these advantageous properties, they can be applied in vibration dampers, in elastic support of smaller mechanical devices and systems, as well as efficient actuators (artificial muscles) in mechatronic and prosthetic devices. Current publications present results of research in the field of nylon spring production, measurement and improvement of their mechanical and thermomechanical properties, as well as methods of their thermal activation.

In [1], a novel type of coiled polymer actuator is demonstrated under thermal stimulation realized via microwave radiation. The authors of [38] proposed a novel and facile methodology

*Corresponding author. Tel.: +421 911 476 101, e-mail: justin.murin@stuba.sk.
<https://doi.org/10.24132/acm.2026.1091>

to chemically construct a thin and highly compliant metallic electrode onto a twisted and coiled nylon-6 fibre (TCN) with a three-dimensional structure via surface modification of the TCN eliciting gold-sulphur (Au–S) interaction for enabling durable electro-thermally induced actuation performance of a TCN actuator (TCNA). In [10], the tensile deformation of chicken muscle fibres and the temperature dependence of the tensile deformation of non-twisted nylon-6 lines and twisted nylon-6 (artificial muscles) is studied. In [28], the graphene materials with high thermal conductivity and low thermal mass were used as coating materials on a nylon twisted and coiled actuator (TCA) surfaces to enhance its dynamic responses. In the study [26], the training effect is investigated for a twisted and coiled nylon yarn heated over two temperature ranges (25–50 °C and 50–75 °C). The training effect was most obvious in the lower temperature range, but nearly absent in the higher temperature range. In [2], two types of coiled actuators were fabricated by inserting certain numbers of twists to polyamide-6 (PA6) single fibres and yarns. The effects of twisting numbers and precursors type on actuation responses, physical properties, and structural changes were investigated. It was demonstrated that fibre-based and yarn-based actuators had different actuation properties. The former one showed a greater actuation load and the latter one had a faster torsional actuation. The paper [29] provides a comprehensive overview of the principles, fabrication strategies, and materials characterization methods of twisted and coiled polymer actuators (TCPAs), including thermomechanical, structural, and morphological analysis at different length scales. To achieve high durability and excellent mechanical performances of cementitious materials, using of the TCPA is proposed and studied in [12]. The employed TCPA has a structure of coiled nylon fibres with wrapping stainless-steel fibres. In [3], the effect of draw ratio on the torsional actuation performances of twisted nylon-6 fibre from an experimental and theoretical perspective is investigated. The paper [33] explores solving the asymmetric, slow cooling contraction response of single TCPAs by utilizing an antagonistic pair. The paper [39] reports that simple plying of two or more twisted fibres produces a coiled muscle with a high spring index. This approach could lead to a cost-effective way to create artificial muscles at a large scale.

Considerable attention is also devoted to computational modelling of the action response of nylon devices to mechanical and thermal loading. In [37], a top-down, multi-scale framework to model the thermo-mechanical actuation of twisted-then-coiled polymeric artificial muscles is established. By linking the macroscopic helical spring, mesoscopic cylindrical laminate and microscopic polymer chains, it precisely predicts the stroke and forces of these actuators. The paper [34] discusses a new model and experimental characterization for thermal responses of straight twisted polymer fibre (TPF) actuators in blocked torsion. The proposed model is derived from combining the fibre stiffness matrix with the Shafer's model of free torsion. The paper [13] refers to key research on twisted and coiled polymer artificial muscles (TCPAMs) and sensors, which use low-cost polymer fibres (such as standard fishing line) to create advanced soft robotics and wearable devices. The paper [30] explores the use of super-coiled transparent polymer fibres specifically common nylon and fluorocarbon fishing lines as low-cost, flexible optical waveguides. In [11], a new truss-type deployable manipulator actuated by a parallel twisted and coiled nylon fibre actuator (PTCNYA) is proposed. The deployment experiment, trajectory experiment, and grasping experiment are conducted under the physical prototype to show the feasibility of the proposed manipulator is presented. In [14], a fascicular module consisting of nylon twisted actuators in parallel connection was proposed and a structural configuration method aiming to realize variable stiffness function was presented and corroborated. The paper [36] establishes a physics-based model for predicting the actuation performance of TCPAMs. The work [31] presents the development of a numerical model to

describe the physical behaviour of a supercoiled polymer (SCP) fibres during thermal actuation and describes the process of optical heat deployment in SCP used as a waveguide in the dual actuator-sensor SCP system. In [5], an artificial muscle with large tensile stress and high stability is presented. The review [32] outlines the recent progress in twisted and coiled artificial muscles and their diverse applications in soft robots.

Theoretical and applied research on nylon springs with negative thermal expansion and compressive prestress is also carried out at the authors' workplace. The article [24] presents original results of research on the dampers with passive and semi-active damping using polymer springs with negative thermal expansion. A universal device for measuring the stiffness of nylon springs and the method of measuring the stiffness of nylon tension springs is presented in [7]. The work [15] explores the fundamental thermo-mechanical properties of TCPAs. In [25], a mathematical-physical model for thermoelastic linear and nonlinear analysis of the torsional actuator is compiled, yielding the values of rotation and the actuator's torque. The article [23] presents the results of research on the dampers with passive and semi-active damping using polymer springs with negative thermal expansion. In [19], the results of a structural linear analysis of new dampers utilizing nylon springs are presented. In [16], a geometrically nonlinear finite element of the link type was formulated without any linearization and utilized for computing the large elastic elongation of a metal rod. The incremental solution exhibited superquadratic convergence. An analysis of twisted polymeric fibres by X-ray diffraction is presented in [27]. The results of the nonlinear thermoelastic analysis of nylon springs with negative thermal expansion were presented in a short conference paper [20], in which the solution was obtained semi-analytically.

For computational geometric nonlinear elastic and thermoelastostatic analysis of the nylon springs with negative thermal expansion, a custom geometrically nonlinear NTCS finite element is established in [22]. The motivation for this activity was that commercially available finite element software does not include a suitable geometrically nonlinear finite element for a prestressed tension spring with negative thermal expansion. Modelling nylon springs using 3D solid finite elements is also challenging because the required properties of the twisted nylon fibre are difficult, if not impossible, to measure. This custom straight, two-node link finite element is based on the updated Lagrangian formulation (ULF) at finite displacements. The stiffness matrix of the spring finite element contains both linear and nonlinear parts. In [22], this finite element was originally applied to the computational modeling and simulation of a single-spring artificial muscle and a vibration damper with a parallel spring arrangement. This custom finite element can be used for the modelling and dimensioning of artificial muscles and special thermoelastic actuators, as well as for designing flexible energy storage units for small stationary or mobile systems made of nylon springs with negative thermal expansion and large extension of the springs. The conference paper [6] presents the measurement results of the coefficient of thermal expansion for a chosen nylon spring, which was conducted in a thermal chamber. In the conference paper [8], we focused on the identification of the dynamic parameters of nylon springs using an experimental method. The springs were loaded with a gravitational mass and a vibration test based on the linear stiffness of the springs was performed. A dynamic linear system consisting of one and two springs was also modelled in the commercial software MSC.ADAMS. A comparison of the natural frequencies of the attached gravitational mass revealed significant discrepancies between the measurement and the computation. As might be expected, these discrepancies arose due to an inaccurate computational model that neglected both geometric nonlinearity and the behaviour of the springs when overcoming their initial prestress. The effect of heating on the deformation and natural vibration of the springs was not

considered. The short conference paper [18] deals with geometric nonlinear modeling and simulation of the nylon spring. Ref. [17] discloses a utility model of a device for measuring nylon springs extension.

Based on our experience in the geometrically nonlinear modelling and simulation of a single nylon spring, we presented our research on a V-actuator at the CM2025 scientific conference. Partial results of this research are published in a short conference paper [21]. The present paper represents a significant extension of that conference paper.

2. Background of the study

The natural circular frequency, natural frequency and period of free vibration of a single spring loaded by attached external concentrated gravitational mass m (with a single degree of freedom) can be calculated from the static spring deflection caused by this static gravitational loading. Given the low weight of nylon, the self-weight of the spring can be neglected. Twisting the nylon fishing line creates a tensile, nonlinearly stiffening spring with a compressive prestress force F_p , which in the unloaded state at room temperature compresses the coils of the spring against each other. To stretch it, it is necessary to load the spring, e.g., with a gravitational tensile force $F_m = mg$ greater than its prestress force F_p , where g is the gravitational acceleration. Fig. 1 shows the spring with an initial length l_0 , which changes to l_m after its elastostatic elongation v_m under the gravitational force F_m .

After being heated by ΔT , the spring is shortened by the thermal stroke $\delta_T = k_T \Delta T$ to the length l_{mT} due to the thermomechanical actuation force $F_T = k_m k_T \Delta T$, where k_T is the negative thermal expansion coefficient, and k_m is the measured linear tensile stiffness of the spring. The coefficient of thermal expansion of the physical nylon spring depends on the magnitude of the tensile force F_m and must be determined by measuring its thermal contraction. The thermoelastic static deflection of the spring is $v_{mT} = v_m - \delta_T$, which is a function of the elongation v_m , force F_m , stiffness k_m , and temperature rise ΔT . From the condition $v_{mT} \leq v_m$, the allowable temperature rise ΔT for the given spring parameters and loadings can be determined. This procedure is applicable to linear deformation analysis for small spring extensions.

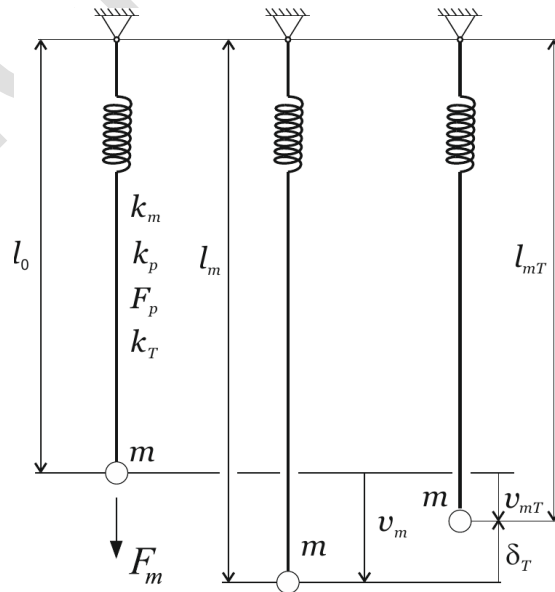
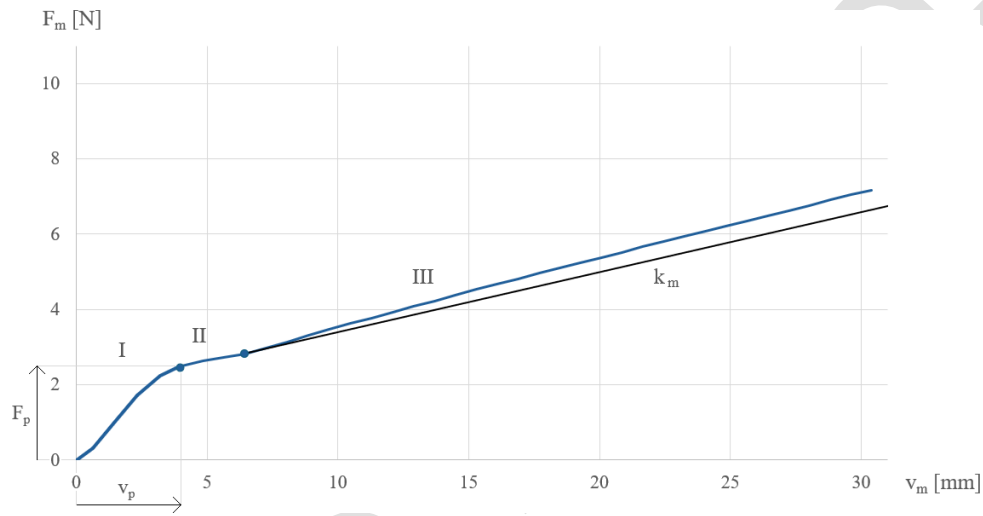


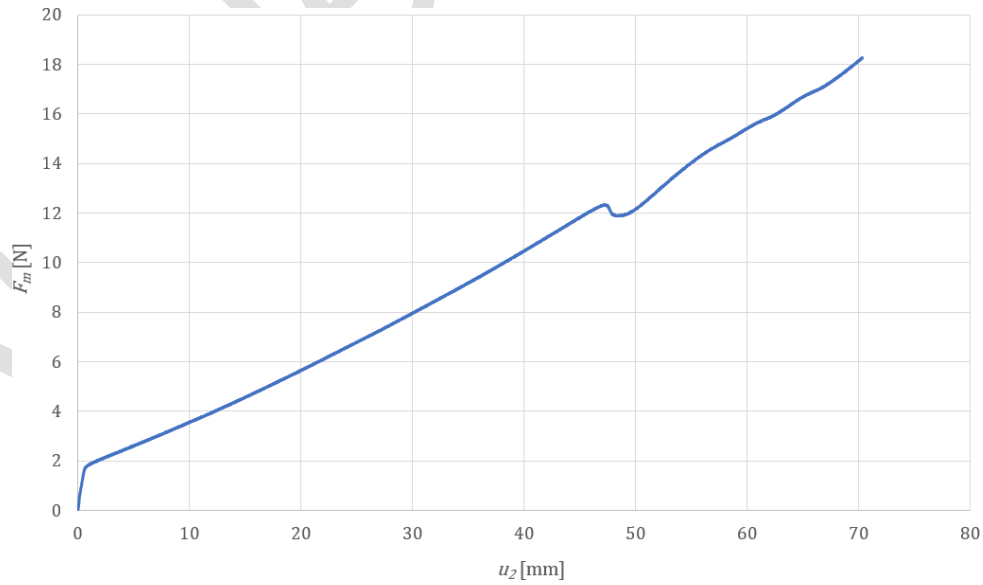
Fig. 1. Elastic elongation and thermal shortening of the nylon spring [18]

At large extensions of the spring, it is necessary to update it according to the geometrically nonlinear theory of continuum mechanics. This update incorporates an incremental method for calculating the large static actuator deflection resulting from significant spring extensions and contractions caused by both the gravitational force of the attached mass and thermal heating. Fig. 2 shows the measured tensile characteristics of the nylon springs loaded with a continuously increasing force F_m . Fig. 2a shows a tensile test of a nylon spring, whose initial length of $l_0 = 85$ mm is extended by 26 %. Fig. 2b shows a tensile test for the nylon spring with an initial length of $l_0 = 40$ mm, which is extended by 118 %. After such a large extension, the coils of the shorter spring begin to unwind. During continuous unloading after reaching the permissible load limit, the spring's elongation shortens without significant hysteresis.

A comparison of the measured tensile characteristics shows that the geometric nonlinearity



(a)



(b)

Fig. 2. Measured tensile characteristics of the single nylon springs loaded with the mechanical force F_m , with the initial length: (a) $l_0 = 85$ mm, (b) $l_0 = 40$ mm [22]

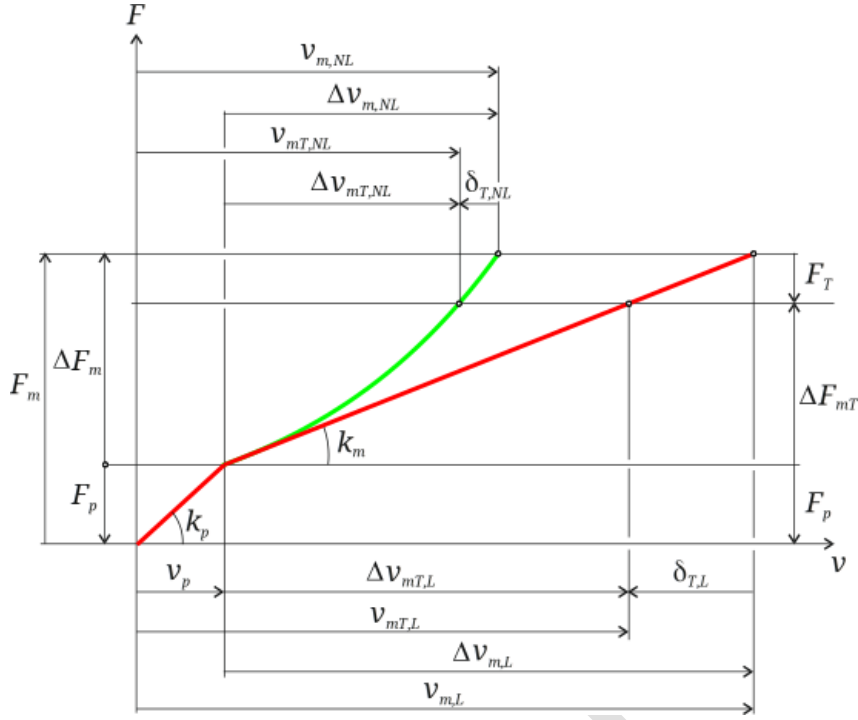


Fig. 3. Computational diagram of the spring tensile test for linear (red line) and nonlinear (green line) calculation model loaded by the gravitational F_m and thermomechanical F_T forces

of the springs becomes significant at large elongations. However, the nonlinearity is already noticeable at a smaller relative elongation of a longer spring, which is shown in Fig. 2a. The reduction of the mechanical elongation of the spring caused by the constant loading force F_m also activates its heating. These tensile tests were performed on the authors' custom measuring device [7]. As a basis for the computational analysis of the deformation response of the spring to its total mechanical and thermal loads, F_m and F_T , the simplified equivalent diagram of the tensile test shown in Fig. 3 is considered. The linear behaviour is shown in red and the nonlinear behaviour is shown in green in this figure. The horizontal axis of Fig. 3 shows the total deformation quantities of the linear (index L) and nonlinear (index NL) calculation models.

The left side of this diagram represents mechanical elastic loading, whereas the right side represents both the mechanical and thermomechanical loading of the spring. The measured and equivalent diagrams consist of three parts, denoted in Fig. 2a by I, II, and III. In the first part (I), which is linearised in Fig. 3, the loading force overcomes the negative spring prestress F_p , thereby elongating the spring by the value v_p . In this state, the effective axial force in the spring is equal to zero. From these measured quantities, the initial linearized spring stiffness $k_p = F_p/v_p$ can be determined with sufficient accuracy. The magnitude of the elongation v_p increases with the initial spring length l_0 (due to the increasing number of the compressed spring coils). For the shorter spring depicted in Fig. 2b, this elongation is negligible. However, as shown in Fig. 2a, for longer springs, the elongation is more significant, and must therefore be considered in the calculation models. After overcoming the initial prestress, the spring begins to elongate linearly in the second part (II) of the tensile characteristics. From this short region, the linear spring stiffness k_m can be determined. In the third region (III), the spring behaves as a nonlinearly stiffening spring. The spring stiffness increases with its increasing elongation (with increasing tensile force), and it is geometrically nonlinear. If the spring loaded with the

gravitational force F_m is heated by a temperature difference ΔT , a thermomechanical actuation force $F_T = k_m k_T \Delta T$ arises in the spring, which reduces the mechanical elongation of the spring and lifts the suspended concentrated mass m (corresponding to the gravitational force F_m) by the value δ_T . The resulting static deflection caused by both the resulting mechanical F_m and thermomechanical F_T forces is v_{mT} . For a linear single spring, $v_{mT,L} = v_p + \Delta v_{mT,L}$, where $\Delta v_{mT,L} = (F_m - F_T)/k_m$. As described in [18], the resultant nonlinear static deflection $\Delta v_{mT,NL}$ is calculated by incremental Newton method and the NTCS finite element.

The negative thermal expansion coefficient k_T for a given mechanical force F_m can be determined experimentally and computationally according to the method described and applied in the articles [6,20,22]. Authors' measurements revealed that the magnitude of this coefficient for the measured spring decreases quasilinearly with increasing force F_m . At a certain magnitude of the force $F_m = F_{m,max}$, the spring no longer shortens due to heating because k_T approaches zero. As indicated in [8], which deals with elastic vibration of the nylon springs, geometric nonlinearity and even the initial elongation v_p are not considered in existing computational models. This simplification subsequently leads to significantly different results between measurements and computational simulations. For this reason, this article focuses on a complex computational study of the influence of the geometric nonlinearity and heating on the parameters of the free natural vibration of the nylon artificial muscles. When calculating the resulting static deflection of the studied actuator, the displacement v_p , caused by overcoming the negative spring prestress F_p , is also considered. The issue of the influence of geometric nonlinearity and heating on the natural frequency of a single-spring actuator was addressed in the authors' work [18]. That study confirmed the significant influence of these factors on the natural vibration of such simple single-spring artificial muscle, which was also validated by measurements.

From the overview of the current literature concerning the issue of mathematical-computational analyses of the nylon actuators natural frequencies at large static extensions and shortenings of the springs. However, nylon spring systems can be subjected to larger static or harmonic mechanical or thermomechanical loading in practical applications. Knowledge of their thermal stroke and natural free vibration parameters is therefore essential for their design and safe operation. These considerations motivated us to leverage our experience with the geometrically nonlinear modeling of single-spring nylon actuators to develop multi-spring configurations, aiming to enhance their functionality and actuation efficiency.

In the present paper, a single-degree-of-freedom thermomechanical actuator is proposed, which is assembled using two V-shaped nylon springs (V-actuator). Considering the prescribed boundary conditions (2), its sole degree of freedom is the displacement of the actuation point in the direction of the gravitational force exerted by the attached external concentrated mass. The resulting static deflection is induced by the gravitational force of this attached mass as well as by the thermomechanical force arising from the heating of the springs (thermal stroke). The custom NTCS finite element [22] is used for the geometrically nonlinear deformation analysis of the actuator. Based on this resulting static deflection of the actuator, the natural frequency of the undamped oscillation of the attached mass is determined. To calculate the static deflection of the V-actuator, the measured parameters of the physical springs are considered. The influence of geometric nonlinearity and heating on the actuator's thermal stroke and natural frequency of the oscillating mass is investigated. The low self-weight of the springs is neglected. Since the resonant frequency of the actuator is investigated, spring damping is not included in the computational analyses.

m_a up, maximally to the initial configuration of the actuator (the blue-coloured configuration in Fig. 4b). Based on the ULF, its current reference configuration is located at time t (the red-coloured configuration in Fig. 4b). According to Figs. 4b and 6, the increment in the mechanical and thermal loads of the actuator,

$$\Delta F_{an} = \frac{\Delta F_a}{nmax} = \frac{F_a - (F_{pa} + F_{Ta}) \frac{h_0}{l_0}}{nmax} = \frac{F_a - 2(F_p + F_T) \frac{h_0}{l_0}}{nmax},$$

causes a displacement increment ${}^t\Delta v = \Delta F_{an}/{}^tk_{ma,NL}$ at point 1 from the reference configuration t to the current configuration at time $t + \Delta t$. In this current configuration, the vertical displacement at node 1 is ${}^{t+\Delta t}v = {}^tv + {}^t\Delta v$. The magnitude of the load increment depends on the selected number of load steps $nmax$. In this study, the same increment magnitude is considered for all load steps. $F_T = k_m k_T \Delta T$ is the resultant thermomechanical force in each spring due to heating ΔT , where k_T is the negative thermal expansion coefficient that must be obtained experimentally. According to Fig. 4b, the springs and actuator reference parameters at time t are the vertical displacement ${}^tv$, the springs length ${}^tl$, the height th , the effective axial force ${}^tN$, the tensile vertical nonlinear stiffness of the actuator ${}^tk_{ma,NL}$, and the spring angles ${}^t\alpha_1$ and ${}^t\alpha_2$. The reference configuration of the actuator gradually changes during the incremental geometric nonlinear solution method up to the position caused by the total magnitude of the gravitational F_a and temperature F_T forces. A procedure for establishing the incremental equations for calculating the increments of the primary quantities (nodal displacements and reaction forces) of the V-actuator is briefly presented in Section 3.1. In the descriptions above, the time designation has only a formal meaning, serving only to indicate the position of the deformed spring configuration during individual loading steps.

When $\Delta T = 0$, the deformation of the actuator caused solely by the gravitational force F_a is investigated. If $nmax = 1$, a purely linear response of the actuator to the applied gravitational, or thermomechanical load is obtained. The resultant linear static deflection of the actuator is denoted by the symbol $v_{a,L}$ in the following calculations. For $nmax > 1$, the resultant geometrically nonlinear static vertical displacement $v_{a,NL}$ of actuator node 1 is obtained. The convergence of the incremental method for solving the geometrically nonlinear static deflection depends on the number of load steps.

When the resultant static equilibrium deflection ${}^{t+\Delta t}v \equiv v_a$ at node 1 (after $nmax$ load steps) of the V-actuator is known, the angular frequency of the natural undamped small oscillation of the external mass m_a suspended on the actuator can be determined. In the calculation of the natural angular frequency of the actuator, the oscillation amplitude of the attached mass m_a around the resultant static equilibrium deflection $v_{a,L}$ or $v_{a,NL}$ is assumed to be small. For an actuator with a single degree of freedom, the angular frequency of free natural vibration around the nonlinear statical deflection $v_{a,NL}$ is $\omega_{0,NL} = \sqrt{k_{a,ef}/m_a}$, where the effective vertical stiffness of the actuator is $k_{a,ef} = m_a g / v_{a,NL}$. The nonlinear static deflection includes the nonlinear relationship between the applied gravitational force $F_a = m_a g = k_{a,ef} v_{a,NL}$ and the equilibrium geometrically nonlinear static deflection of the actuator $v_{a,NL}$. Hence, $k_{a,ef} = m_a g / v_{a,NL}$ and then $\omega_{0,NL} = \sqrt{g/v_{a,NL}}$. As shown by the tensile test of the actuator in Fig. 5, the effective stiffness of the actuator ($k_{a,ef}$) should comprise three different stiffnesses. Determining its discrete value is practically very difficult, if not impossible. But the geometrically nonlinear static deflection ($v_{a,NL}$) of the actuator for the resulting actuator load is determined using a geometrically nonlinear computational model, which incorporates the entire tensile characteristic of the actuator. For this reason, the natural angular frequency in this study is determined from static displacements $v_{a,NL}$ and $v_{a,L}$.

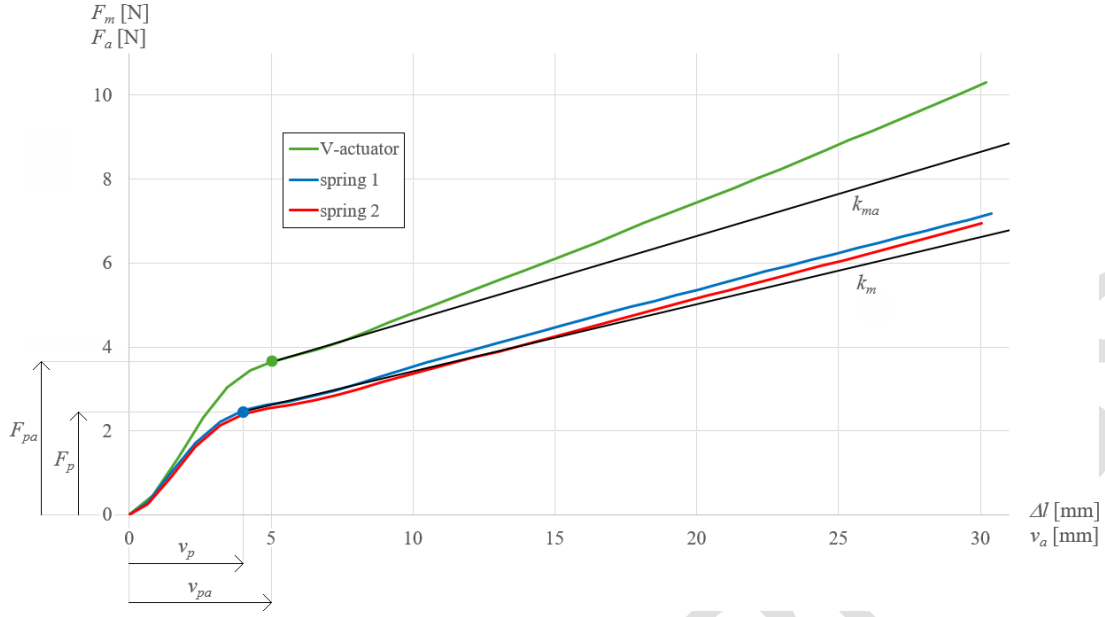


Fig. 5. Measured tensile diagrams of the springs with a length $l_0 = 85$ mm and the V-actuator with a span $a = 51.13$ mm, loaded by mechanical forces F_m and F_a [21]

3.1. Establishment of the geometrically nonlinear incremental equations to determine the elastostatic and thermoelastostatic deflections required for the thermal stroke and the natural frequency calculation of the V-actuator

The procedure for establishing the incremental equations for calculating the increments of the primary quantities (nodal displacements as well as reaction forces, if needed) of the V-actuator is based on ULF and FEM. Each spring of the actuator is assumed to be a two-node NTCS link-type finite element [22] with two degrees of freedom at each nodal point in the planar global coordinate system (x, y) . The nonlinear global stiffness matrix of the finite elements No. 1 and No. 2 in the planar global coordinate system has a dimension of 4×4 . After their extension, these stiffness matrices have a dimension of $\text{NDOF} \times \text{NDOF}$, where $\text{NDOF} = 6$ is the total number of degrees of freedom of the three nodal points of the actuator. The process of constructing the extended global nonlinear stiffness matrix $[{}^tK_{a,NL}]$ of this actuator is standard within the FEM procedure. The nonlinear stiffness matrix contains a linear and a geometric part. It should be noted that the linear stiffness matrix is not modified during the incremental solution process. Only the geometric stiffness matrix is updated. The resulting general global FEM equations of the actuator for calculating the unknown displacement increments and coupling reactions for the common time step Δt has the following form:

$$[{}^tK_{a,NL}] \begin{bmatrix} {}^t\Delta u_1 \\ {}^t\Delta v_1 \\ {}^t\Delta u_2 \\ {}^t\Delta v_2 \\ {}^t\Delta u_3 \\ {}^t\Delta v_3 \end{bmatrix} = \begin{bmatrix} {}^t\Delta R_{11} \\ \Delta F_{12} \\ {}^t\Delta R_{21} \\ {}^t\Delta R_{22} \\ {}^t\Delta R_{31} \\ {}^t\Delta R_{32} \end{bmatrix}. \quad (1)$$

On the left-hand side of (1), in addition to the nonlinear stiffness matrix of the actuator $[{}^tK_{a,NL}]$ at the reference configuration (at time t), there are the displacement increments of

nodes 1–3 in the directions of the global x - and y -axes (denoted by the letters u and v , respectively). On the right-hand side of (1), there are the increments of the coupling reactions at nodes 2 and 3 (denoted by the symbol ΔR) in the direction of the x -axis (index 1), and the y -axis (index 2). The term $\Delta F_{12} = \Delta F_{an}$ is the increment of the load at node 1 (index 1) in the direction of the y -axis (index 2). After applying the boundary conditions

$${}^t\Delta u_1 = {}^t\Delta u_2 = {}^t\Delta u_3 = {}^t\Delta v_2 = {}^t\Delta v_3 = 0, \quad \Delta R_{11} = 0, \quad (2)$$

the reduced system

$$[{}^tK_{a,NL}] \begin{bmatrix} 0 \\ {}^t\Delta v_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \Delta F_{an} \\ {}^t\Delta R_{21} \\ {}^t\Delta R_{22} \\ {}^t\Delta R_{31} \\ {}^t\Delta R_{32} \end{bmatrix} \quad (3)$$

is obtained for calculating the increment of the vertical displacement of node 1 (${}^t\Delta v_1$) and the increments of the coupling reactions at nodes 2 and 3.

In (3), the global nonlinear stiffness matrix of the actuator is the sum of the global nonlinear stiffness matrices of finite elements No. 1 and No. 2 in the reference configuration t : $[{}^tK_{a,NL}] = [{}^tK_{1,NL}] + [{}^tK_{2,NL}]$. The global nonlinear stiffness matrix of finite element No. 1 at the reference configuration is equal to the sum of the extended linear and geometric matrices: $[{}^tK_{1,NL}] = [K_{1,L}] + [{}^tK_{1,G}]$. The extended linear stiffness matrix of the finite element No. 1 reads

$$[K_{1,L}] = k_m \begin{bmatrix} c_1^2 & s_1 c_1 & -c_1^2 & -s_1 c_1 & 0 & 0 \\ s_1 c_1 & s_1^2 & -s_1 c_1 & -s_1^2 & 0 & 0 \\ -c_1^2 & -s_1 c_1 & c_1^2 & s_1 c_1 & 0 & 0 \\ -s_1 c_1 & -s_1^2 & s_1 c_1 & s_1^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

where $s_1 = \sin \alpha_{10} = h_0/l_0$, $c_1 = \cos \alpha_{10} = a/l_0$, and k_m is the linear stiffness of the spring. The extended geometric stiffness matrix of the finite element No. 1 reads

$$[{}^tK_{1,G}] = \frac{{}^tN_1}{t l_1} \begin{bmatrix} {}^t c_1^2 & {}^t s_1 {}^t c_1 & -{}^t c_1^2 & -{}^t s_1 {}^t c_1 & 0 & 0 \\ {}^t s_1 {}^t c_1 & {}^t s_1^2 & -{}^t s_1 {}^t c_1 & -{}^t s_1^2 & 0 & 0 \\ -{}^t c_1^2 & -{}^t s_1 {}^t c_1 & {}^t s_1 {}^t c_1 & {}^t s_1^2 & 0 & 0 \\ -{}^t s_1 {}^t c_1 & -{}^t s_1^2 & {}^t s_1 {}^t c_1 & {}^t s_1^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

where ${}^t c_1 = \cos {}^t \alpha_1 = {}^t h/t l$, ${}^t s_1 = \sin {}^t \alpha_1 = {}^t a/t l$. The term ${}^t N_1$ is the effective axial force in the finite element No. 1 with the length ${}^t l_1$ at the reference configuration t .

The global nonlinear stiffness matrix of the finite element No. 2 at the reference position is equal to the sum of the extended linear and geometric matrices: $[{}^tK_{2,NL}] = [K_{2,L}] + [{}^tK_{2,G}]$.

The extended linear stiffness matrix of finite element No. 2 has the following form:

$$[K_{2,L}] = k_m \begin{bmatrix} c_2^2 & s_2 c_2 & 0 & 0 & -c_2^2 & -s_2 c_2 \\ s_2 c_2 & s_2^2 & 0 & 0 & -s_2 c_2 & -s_2^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -c_2^2 & -s_2 c_2 & 0 & 0 & c_2^2 & s_2 c_2 \\ -s_2 c_2 & -s_2^2 & 0 & 0 & s_2 c_2 & s_2^2 \end{bmatrix},$$

where $s_2 = \sin \alpha_{20}$, $c_2 = \cos \alpha_{20}$. The extended geometric stiffness matrix of the finite element No. 2 at the reference position reads

$$[{}^t K_{2,G}] = \frac{{}^t N_2}{{}^t l_2} \begin{bmatrix} {}^t c_2^2 & {}^t s_2 {}^t c_2 & 0 & 0 & -{}^t c_2^2 & -{}^t s_2 {}^t c_2 \\ {}^t s_2 {}^t c_2 & {}^t s_2^2 & 0 & 0 & -{}^t s_2 {}^t c_2 & -{}^t s_2^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -{}^t c_2^2 & -{}^t s_2 {}^t c_2 & 0 & 0 & {}^t c_2^2 & {}^t s_2 {}^t c_2 \\ -{}^t s_2 {}^t c_2 & -{}^t s_2^2 & 0 & 0 & {}^t s_2 {}^t c_2 & {}^t s_2^2 \end{bmatrix},$$

where ${}^t c_2 = \cos {}^t \alpha_2$, ${}^t s_2 = \sin {}^t \alpha_2$. The term ${}^t N_2$ is the effective axial force in finite element No. 2 with the length ${}^t l_2 = {}^t l_1$ in the reference configuration t . The global nonlinear stiffness matrix of the finite elements No. 1 and No. 2 at the reference position is the sum of its linear and geometric matrices: $[{}^t K_{1,NL}] = [K_{1,L}] + [{}^t K_{1,G}]$ and $[{}^t K_{2,NL}] = [K_{2,L}] + [{}^t K_{2,G}]$. After performing the appropriate mathematical operations, the resulting equation

$${}^t \Delta v_1 = \frac{\Delta F_{12}}{2 \left[k_m \left(\frac{h_0}{l_0} \right)^2 + \frac{{}^t N}{{}^t l} \left(\frac{{}^t h}{{}^t l} \right)^2 \right]} \equiv {}^t \Delta v = \frac{\Delta F_{an}}{{}^t k_{ma,NL}} \quad (4)$$

is obtained from (3) for calculating the unknown increment of the vertical displacement of node 1 in the current loading step.

In (4), $\Delta F_{12} = \Delta F_{an}$ is the above-mentioned load increment for each loading step, and $2 \left[k_m \left(\frac{h_0}{l_0} \right)^2 + \frac{{}^t N}{{}^t l} \left(\frac{{}^t h}{{}^t l} \right)^2 \right] = {}^t k_{ma,NL} = {}^t k_{ma,L} + {}^t k_{ma,G}$ is the vertical nonlinear stiffness of the actuator in the respective reference configuration, consisting of the sum of the linear ${}^t k_{ma,L} = 2k_m \left(\frac{h_0}{l_0} \right)^2$ and geometric ${}^t k_{ma,G} = 2{}^t N / {}^t l \left(\frac{{}^t h}{{}^t l} \right)^2$ vertical stiffnesses. The notation of ${}^t \Delta v_1$ is simplified to the notation of ${}^t \Delta v$ in (4). Using the calculated vertical displacement increment of node 1, the coupling reaction increments at the nodes 2 and 3 can be calculated from (3). Subsequently, the increments of the spring extension and the effective axial forces in the springs, as well as the geometric parameters of the actuator in the current configuration $t + \Delta t$ are determined. For instance, the vertical displacement at point 1 in the instant configuration $t + \Delta t$ is ${}^{t+\Delta t} v = {}^t v + {}^t \Delta v$, the actuator height is ${}^{t+\Delta t} h = {}^t h + {}^{t+\Delta t} v$, the length of the springs is ${}^{t+\Delta t} l = \sqrt{a^2 + {}^{t+\Delta t} h^2}$, the extension of the springs is ${}^{t+\Delta t} \Delta l = {}^{t+\Delta t} l - l_0$, and the current effective axial force in the springs is ${}^{t+\Delta t} N = (k_m + {}^t N / {}^t l) {}^{t+\Delta t} \Delta l$. These actuator parameters at time $t + \Delta t$ will serve as the reference parameters at time t for the next loading step. The incremental solution of (4) is carried out over $i = 1, n_{max}$ loading steps. Analogous to the solution of the static deflection of a single spring [18], the above incremental method for solving the nonlinear actuator equation must be updated for the first load step ($i = 1$) due to nonlinear shape of the measured tensile characteristic of the springs to include its first part (I), see Fig. 2a.

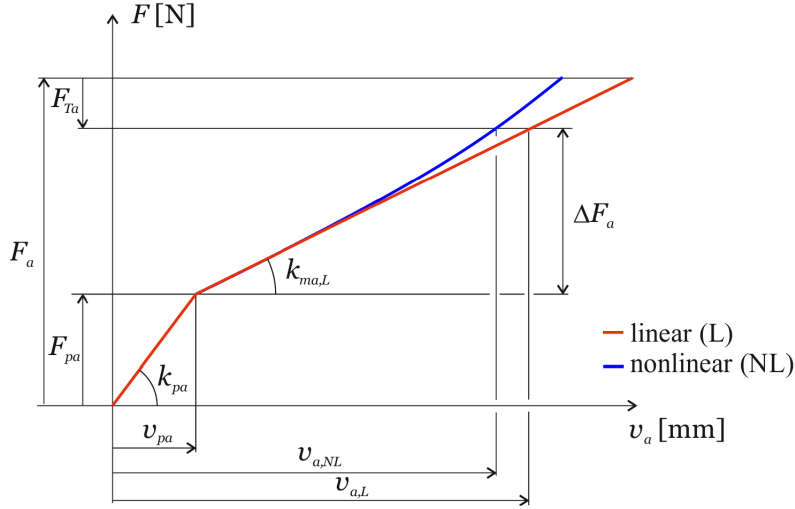


Fig. 6. Simplified tensile diagram of the V-actuator for the computational linear and nonlinear models for thermoelastic loading

Fig. 5 shows the measured tensile characteristics of two individual springs of the same length $l_0 = 85$ mm (represented by the blue and red lines) loaded with a continuously increasing force F_m . From this diagram, a high consistency between the springs is evident. The physical V-actuator is assembled from these two springs with a spring span $a = 51.13$ mm ($\varphi_0 = 37^\circ$). This span was obtained by measuring the physical model of the actuator. The actuator is loaded with a mass m_a , corresponding to the gravitational force $F_a = m_a g$. The measured tensile characteristics of the actuator is shown in Fig. 5 by the green line. Analogous to the measured tensile diagram of individual springs that is shown in Fig. 2a, the measured diagram of the V-actuator also consists of three regions. In the first region, the loading force overcomes the initial compressive prestress F_{pa} . At the end of this part of the diagram, the effective axial force in the springs is equal to zero, because the current tensile loading force at this point equals the negative prestress of the actuator F_{pa} . The corresponding displacement is denoted as v_{pa} . By linearly extending the short second section of the tensile diagrams in Fig. 5, the linear stiffness lines of the individual springs as well as that of the actuator are obtained. The black lines in Fig. 5 determine the linear stiffness of a single spring k_m and the linear stiffness of the actuator k_{ma} . In the third region, the measured stiffness of the actuator increases nonlinearly. As expected, the stiffness of the actuator with the considered span a is greater than the stiffness of the individual springs. This arrangement of springs allows for greater action intervention of the V-actuator.

A simplified diagram of the tensile test of the actuator loaded with the gravitational force F_a is shown in Fig. 6. Analogously to the simplified tensile diagram of a single spring shown in Fig. 3, the simplified tensile diagram of the actuator is characterized by three regions. In the first region, the loading force overcomes the initial compressive prestress in the springs. The measured curve of the tensile diagram of the actuator in Fig. 5 is linearized in Fig. 6 within this first region. In the second and third parts of the simplified tensile diagram (Fig. 6), the red line shows the linear response, while the blue curve represents the nonlinear response of the actuator to the applied load. The vertical displacement of actuator node 1 is denoted as v_a in Fig. 6.

According to Fig. 5 and Section 2, the measured prestress in the single springs is F_p , which corresponds to the measured spring elongation v_p . The linearized stiffness of a single spring in the first part of the tensile diagram is then $k_p = F_p/v_p$ and the linear stiffness is k_m . From

the measured spring parameters, the input parameters of the actuator can be determined by the following expressions: $F_{pa} = 2F_p h_0/l_0$ is its compressive prestress, $k_{pa} = 2k_p(h_0/l_0)^2$ is its linearised initial stiffness, and the initial vertical displacement of node 1 of the actuator is $v_{pa} = F_{pa}/k_{pa} = F_p/(k_p h_0/l_0)$. After overcoming the negative prestress, the individual spring behaves linearly over a short section with the measured linear stiffness k_m . This linearity of the springs is also reflected in the tensile diagram of the V-actuator with its measured or calculated linear stiffness $k_{ma,L} = 2k_m(h_0/l_0)^2$. After this part of the tensile diagram, the individual spring as well as the V-actuator begin to stiffen nonlinearly. As already mentioned, this nonlinearity increases significantly with increasing mechanical elongation springs by the force F_a . To include the first linearized part of the tensile characteristic in the incremental solution method, the reference actuator configuration for the first loading step ($i = 1$) is defined by the effective axial force in the springs ${}^tN = 0$ and the initial displacement of node 1 of the actuator ${}^tv = v_{pa}$. The calculated actuator stiffness for the first loading step is ${}^tk_{ma,NL} \equiv k_{ma,L}$, i.e., it is equal to its linear stiffness, which is then nonlinearly reinforced during the subsequent loading increase. The above procedures for calculating the resultant static deflection and resonant frequency of the V-actuator have been implemented in the software MATHEMATICA [35] within a custom calculation program called *Vnylonactuator*. This program is used to perform the computational analysis of the V-actuator loaded by an attached mass and subjected to heating of the springs. The calculation results are presented in Section 3.2. In all subsequent nonlinear numerical calculations, $nmax = 100$ loading steps are used to obtain convergent results with sufficient accuracy. The convergence of the geometrically nonlinear calculation for the static thermoelastic deflection of the V-actuator, using the *Vnylonactuator* code, is presented in Section 3.2 (Case 3).

3.2. Numerical calculation of the static deflection and natural angular frequency of the V-actuator

The physical model of the V-actuator is assembled from springs, whose measured tension diagrams for $\Delta T = 0$ are shown in Fig. 5. Due to small differences in their tensile characteristics, the spring, whose measured tensile characteristics is marked in blue in Fig. 5, was selected for the computational model of the actuator. These springs were wound on our proprietary device [15] from a nylon line Specialist Catfish with the diameter of 0.7 mm. The simplified equivalent characteristics for the elastic and thermoelastic, linear and nonlinear extension of the nylon spring is shown in Fig. 3. By measuring this spring, the following parameters were determined: the initial spring length $l_0 = 85$ mm, the initial spring stiffness $k_p = 0.61$ N/mm, the initial spring elongation $v_p = 4.01$ mm, the compressional spring prestress force $F_p = 2.45$ N, and the linear spring stiffness $k_m = 0.16$ N/mm.

As the authors' measurements show, these parameters do not change even when the continuously increasing loading is interrupted at a lower or higher force, such as the maximum force F_m shown in Fig. 5. Both the actuator and the springs unload along the same path as during loading without significant hysteresis. The process of continuous spring loading and unloading is repeatable without a significant change in the tensile path. The results of the computational analyses of the V-actuator, focused on the effects of geometric nonlinearity and heating on the deformation and natural frequency under various actuator loadings, are documented in the following five cases (Case 1 to Case 5). In all computational analyses, a spring span of $a = 51.13$ mm is considered, while the magnitude of the static deflections and the angular frequency are compared with an actuator having parallel spring arrangement.

Case 1: Effect of the force magnitude $F_a = m_a g$ on the linear and nonlinear elastostatic deflection and the natural angular frequency of the V-actuator

In Case 1, the results of the computational analysis of the V-actuator are presented to assess the influence of geometric nonlinearity on its natural angular frequency of the oscillating mass. The natural angular frequency of the oscillating mass is determined considering a small displacement from its equilibrium position. The initial length of the springs is $l_0 = 85$ mm and the initial half-angle of spring span $\varphi_0 = 37^\circ$. For this angle, the span and the initial height are $a = 51.13$ mm and $h_0 = 67.9$ mm, respectively. The measured tensile characteristics of the considered springs is shown in Fig. 5. From the above-mentioned quantitative parameters of the selected springs, the following parameters of the actuator are calculated: the prestress force $F_{pa} = 3.914$ N, the vertical displacement $v_{pa} = 5.03$ mm, the initial prestress stiffness $k_{pa} = 0.78$ N/mm, and the linear stiffness $k_{ma,L} = 0.204$ N/mm. These calculated parameters agree well with the results of the measurement of the tensile characteristics of the actuator shown in Fig. 5. The results of the computational simulation for Case 1 are presented in Table 1. The gravitational force $F_a = m_a g$ varies from 3.94 N (slightly larger than the actuator prestress $F_{pa} = 3.914$ mm) to 30 N. At this force, a large static extension of the springs occurs. $\Delta F_a = F_a - 2F_{pa}h_0/l_0$ is the resultant computational load applied around tensile line after overcoming the actuator prestress F_{pa} . The variables v_a , ω_0 , Δl and N are the resultant static deflection, the natural angular frequency, the spring extension and the effective axial force, respectively. The indices L and NL denote the results of the linear and nonlinear simulations, respectively. The index exp denotes the results of the actuator measurements. Fig. 7 shows the chosen results graphically. It is noteworthy to mention that large amplitudes of natural vibration are outside the scope of this article.

As shown in Table 1 and Fig. 7, significant difference between the linear and nonlinear static deflections occurred for larger values of the force F_a . The magnitudes of the measured deflection are in good agreement with the nonlinearly calculated quantities. The linear extension of the springs (Δl_L) significantly exceeds the size of their initial length l_0 already at $F_a = 25$ N.

Table 1. Computational elastostatic and natural frequency analysis results compared with experimental measurements for the V-actuator under various gravitational force magnitudes (F_a) for Case 1

F_a [N]	ΔF_a [N]	$v_{a,L}$ [mm]	$v_{a,NL}$ [mm]	$v_{a,exp}$ [mm]	$\omega_{0,L}$ [rad/s]	$\omega_{0,NL}$ [rad/s]	$\omega_{0,exp}$ [rad/s]	Δl_L [mm]	Δl_{NL} [mm]	N_L [N]	N_{NL} [N]
3.94	0.026	5.15	5.15	6.68	43.64	43.64	38.32	4.17	4.17	0.026	0.025
5.00	1.086	10.34	10.22	10.75	30.80	30.98	30.21	8.47	8.37	0.713	0.66
5.55	1.640	13.04	12.76	12.77	27.43	27.73	27.72	10.74	10.50	1.08	0.99
6.06	2.146	15.08	14.65	14.84	25.50	25.88	25.71	12.47	12.10	1.35	1.24
6.94	3.026	19.40	18.54	18.09	22.49	23.00	23.29	16.17	15.43	1.94	1.75
8.01	4.096	24.64	23.11	22.15	19.95	20.60	21.03	20.72	19.39	2.67	2.37
9.15	5.236	30.22	27.81	26.19	18.02	18.78	19.35	25.64	23.51	3.46	3.02
10.07	6.156	35.17	31.84	29.41	16.70	17.55	18.26	30.60	27.09	4.17	3.60
10.31	6.39	36.35	32.78	30.21	16.39	17.30	18.02	31.11	27.92	4.34	3.73
15.00	11.086	59.32	50.06	–	12.83	14.00	–	52.11	43.56	7.69	6.32
20.00	16.086	83.80	66.74	–	10.82	12.12	–	75.09	59.02	11.37	9.03
25.00	21.086	108.29	82.17	–	9.52	10.93	–	98.46	73.54	15.11	11.69
30.00	26.086	132.77	97.14	–	8.59	10.05	–	122.09	87.78	18.89	14.40

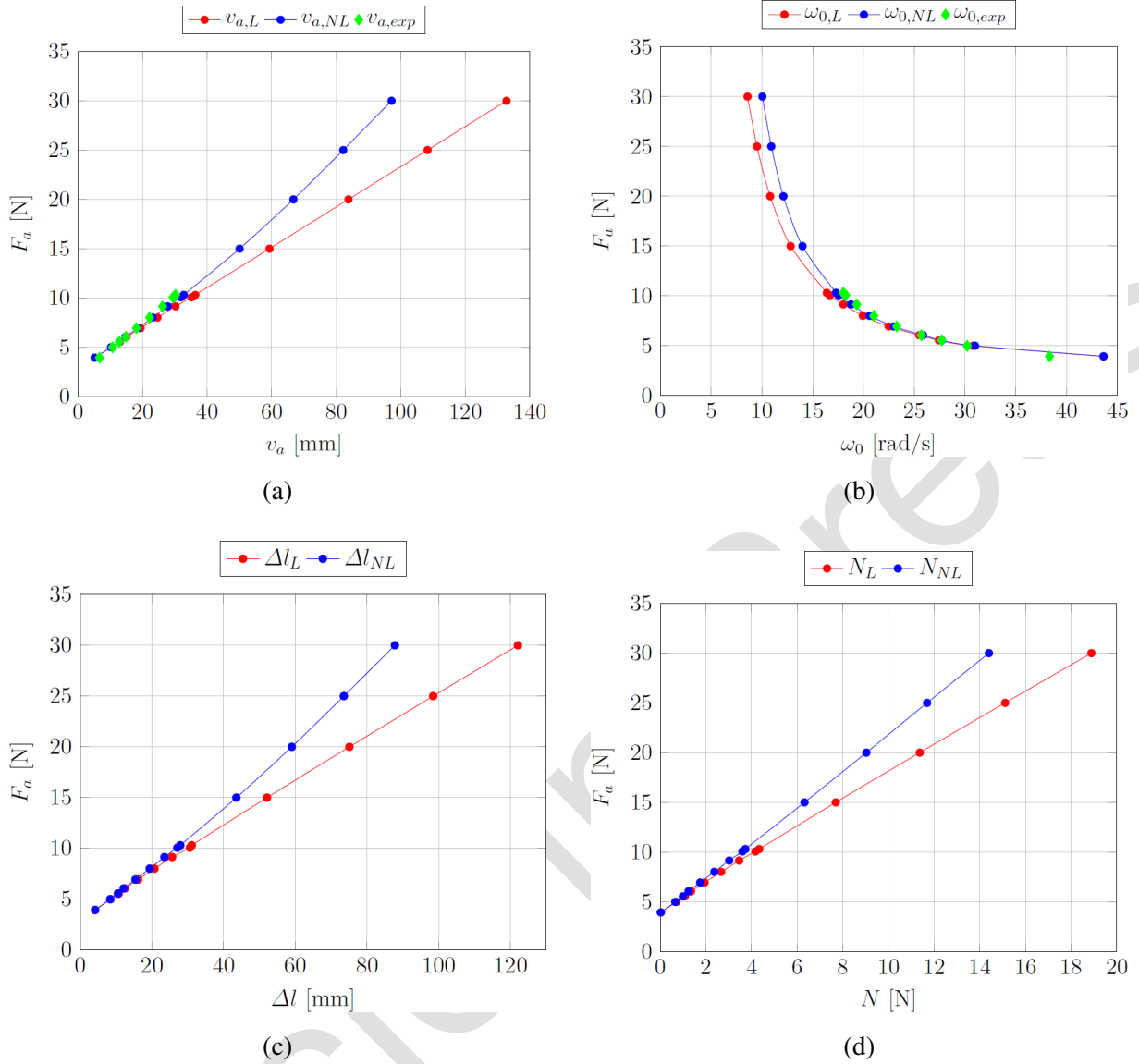


Fig. 7. Calculated and measured quantities as functions of the gravitational force $F_a = m_a g$ for Case 1: (a) static deflection v_a , (b) natural angular frequency ω_0 , (c) spring extension Δl , and (d) effective axial force N

This is therefore an unacceptable state of the actuator. A less significant effect was also observed in the evaluation of the natural angular frequencies. The natural frequency significantly decreases with increasing load (with an increasing external mass m_a). It should be noted that our automatic measuring device allowed us to measure the displacement of point 1 of the actuator to the maximum value of $v_{a,exp} = 30.21$ mm ($F_a = 10.31$ N, $m_a = 1.05$ kg).

Case 2: Analysis of deformation and natural frequency of V-actuator loaded with a force $F_a = 5.55$ N ($m_a = 0.57$ kg) under various heating $\Delta T \in \langle 0, 29 \rangle$ °C

The V-actuator, with the parameters presented in Case 1, is loaded with the force $F_a = 5.55$ N ($m_a = 0.57$ kg), and with the various temperature rise $\Delta T \in \langle 0, 29 \rangle$ °C. The individual springs are elongated with the force $F_m = 3.44$ N. For this magnitude of the force F_m , the measured coefficient of the negative thermal expansion of the springs is $k_T = 0.22$ mm/°C.

The calculated resultant quantities shown in Table 2 are defined as follows: v_a is the ther-

Table 2. Results of elastic ($\Delta T = 0^\circ\text{C}$) and thermoelastic ($\Delta T > 0^\circ\text{C}$) analyses, including the natural angular frequencies of the actuator under constant load $F_a = m_a g = 5.55\text{ N}$ and varying temperature increments $\Delta T \in \langle 0, 29 \rangle^\circ\text{C}$ with $k_T = 0.22\text{ mm}/^\circ\text{C}$ for Case 2

$\Delta T [^\circ\text{C}]$	0	5	10	15	20	22	24	27	29
$F_T [\text{N}]$	0.00	0.18	0.35	0.53	0.704	0.77	0.84	0.95	1.02
$\Delta F_a [\text{N}]$	1.64	1.35	1.07	0.79	0.51	0.40	0.29	0.12	0.005
$v_{a,L} [\text{mm}]$	13.04	11.66	10.28	8.91	7.53	6.98	6.43	5.60	5.05
$sT_L [\text{mm}]$	0.00	1.38	2.76	4.13	5.51	6.06	6.61	7.44	7.99
$v_{a,NL} [\text{mm}]$	12.76	11.47	10.16	8.84	7.50	6.96	6.42	5.60	5.05
$sT_{NL} [\text{mm}]$	0.00	1.29	2.60	3.92	5.26	5.8	6.34	7.16	7.71
$h_L [\text{mm}]$	80.94	79.56	78.18	76.81	75.43	74.88	74.33	73.50	67.9
$h_{NL} [\text{mm}]$	80.66	79.37	78.06	76.74	75.40	74.86	74.32	73.50	72.95
$l_L [\text{mm}]$	95.74	94.58	93.42	92.27	91.13	90.67	90.22	89.54	89.09
$l_{NL} [\text{mm}]$	95.50	94.41	93.31	92.21	91.10	90.66	90.21	89.54	89.09
$\Delta l_L [\text{mm}]$	10.74	9.58	8.42	7.27	6.13	5.67	5.22	4.54	4.09
$\Delta l_{NL} [\text{mm}]$	10.50	9.41	8.31	7.21	6.10	5.66	5.21	4.54	4.08
$\varphi_L [^\circ]$	32.28	32.74	33.20	33.67	34.15	34.34	34.54	34.84	35.04
$\varphi_{NL} [^\circ]$	32.37	32.81	33.04	33.69	34.16	34.85	34.54	34.84	35.04
$N_L [\text{N}]$	0.33	0.89	0.71	0.52	0.34	0.25	0.19	0.08	0.01
$N_{NL} [\text{N}]$	0.99	0.82	0.99	0.49	0.32	0.27	0.18	0.08	0.01
$k_{ma,L} [\text{N/mm}]$	0.204	0.204	0.204	0.204	0.204	0.204	0.204	0.204	0.204
$k_{ma,NL} [\text{N/mm}]$	0.22	0.216	0.214	0.211	0.209	0.207	0.206	0.205	0.204
$\omega_{0,L} [\text{rad/s}]$	27.43	29.00	30.89	33.18	36.09	37.49	39.06	41.85	44.07
$\omega_{0,NL} [\text{rad/s}]$	27.72	29.24	31.07	33.31	36.17	37.54	39.09	41.85	44.07
$\Delta\omega_0 [\text{rad/s}]$	0.29	0.24	0.18	0.13	0.08	0.05	0.03	0.00	0.00

moelastostatic deflection, ω_0 is the natural angular frequency, Δl is the springs elongation, and N is the effective axial force in the springs. $F_T = k_m k_T \Delta T$ is the resultant thermal force in the springs caused by the corresponding heating ΔT , $\Delta F_a = F_a - 2(F_p + F_T)h_0/l_0$ is the computational resultant force acting on the actuator due to the mass m_a and the heating ΔT , acting after overcoming the pressure prestress in the actuator springs. The variable sT is the thermal stroke of the actuator, and l is the length of the springs after the resultant gravitational and thermal loadings. Furthermore, φ is the half-opening angle of the springs, and k_{ma} is the stiffness of the actuator. The thermal stroke sT is determined as the difference between the vertical displacement of point 1 of the actuator due to the tensile force F_a at zero temperature rise ($\Delta T = 0^\circ\text{C}$) and the displacement of this point at the resultant mechanical and thermal force F_m and F_T . Finally, $\Delta\omega_0$ is the difference between the angular frequencies obtained by the linear and nonlinear computational models.

The static deflection of the actuator is measured for temperature increases of $\Delta T = 0^\circ\text{C}$ and 27°C . At zero heating, the elastic deflection $v_{a,exp} = 12.77\text{ mm}$ is measured. This deflection corresponds to the natural angular frequency $\omega_{0,exp} = 27.77\text{ rad/s}$. In the case of heating of $\Delta T = 0^\circ\text{C}$, the thermoelastic deflection $v_{a,exp} = 6\text{ mm}$ is measured, which corresponds to a thermoelastic natural angular frequency of $\omega_{0,exp} = 40.43\text{ rad/s}$. Fig. 8 shows the calculated linear and nonlinear results graphically in red and blue. The measured results are represented

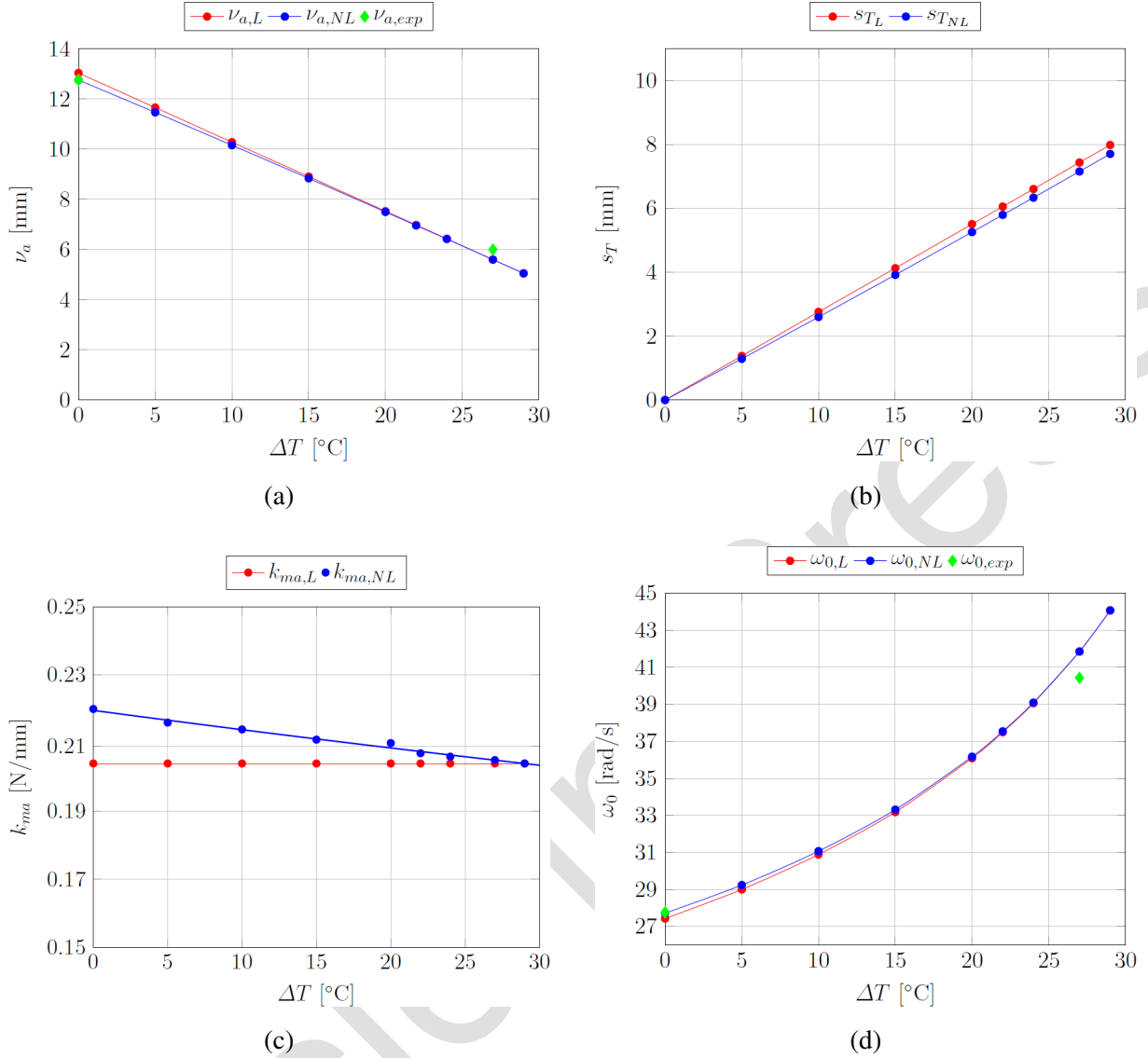


Fig. 8. Comparison of calculated and selected measured quantities for a constant gravitational force of $F_a = 5.55$ N and various heating ΔT for Case 2: (a) static deflection ν_a , (b) thermal stroke s_T , (c) stiffness k_{ma} , (d) natural angular frequency ω_0

by the green-coloured dots in Fig. 8a and 8d.

Based on the obtained calculation results shown in Table 2 and Fig. 8, the following conclusions can be drawn. For the applied gravitational load $F_a = 5.55$ N, a relatively large thermal stroke s_T is achieved at the relatively low temperature rise. The temperature rise of $\Delta T = 29^\circ\text{C} = \Delta T_{\max}$ can be considered as the limit for the selected spring configuration, because the effective axial force in the springs is already close to zero. The heating of the springs significantly increases the natural angular frequency of the actuator. This occurs because at higher spring heating the resultant static deflection ν_a of the actuator significantly decreases. For the relatively small resultant thermoelastic extensions of the springs, the frequency difference between the linear and nonlinear solutions, $\Delta\omega_0$, is logically not significant. Due to the lower tensile load, the coefficient of thermal expansion is large, which results in a larger thermal stroke even with a minor heating. The measured quantities are in satisfactory agreement with the results of both the linear and nonlinear calculation.

To compare the magnitude of the thermal stroke, a calculation was made for an actuator with the span of $a = 0$ mm. For this parallel arrangement of the springs, $\Delta T_{\max} = 8^\circ\text{C}$ and $sT_{NL} = 1.82$ mm. The thermal expansion coefficient is $k_T = 0.23$ mm/ $^\circ\text{C}$. As can be seen from this comparison, the thermal stroke of the V-actuator with a spring span of 51.13 mm is 4.27 times larger.

Case 3: *Elastostatic and thermoelastic deformation analysis and the natural frequency of the V-actuator loaded with a force $F_a = 10.07$ N ($m_a = 1.026$ kg) under various heating $\Delta T \in \langle 0, 140 \rangle^\circ\text{C}$*

The V-actuator with the same parameters as presented in Case 1 is considered. Compared to Case 2, the actuator is loaded with larger force $F_a = 10.07$ N ($m_a = 1.026$ kg) and with various heating $\Delta T \in \langle 0, 140 \rangle^\circ\text{C}$. The individual springs are then elongated with the force $F_m = 6.26$ N. For this value of the force F_m , the coefficient of negative thermal expansion of the springs is $k_T = 0.17$ mm/ $^\circ\text{C}$. The calculated results are shown in Table 3 and selected results are displayed in Fig. 9. The calculated quantities in Table 3 are labelled analogously to those in Table 2.

From a comparison of the calculated results for Case 3 with those from Case 2, the increase in the force F_a was reflected in a greater difference between the displacements and angular frequencies calculated by the linear and nonlinear actuator models. At $\Delta T = 140^\circ\text{C}$, this difference disappears. Despite a smaller thermal expansion coefficient k_T and a larger load m_a , the

Table 3. Results of the elastostatic and thermoelastic deformation analyses and the natural angular frequencies for Case 3 of the V-actuator, loaded with a force of $F_a = 10.07$ N ($m_a = 1.026$ kg) and with various heating $\Delta T \in \langle 0, 140 \rangle^\circ\text{C}$, and $k_T = 0.17$ mm/ $^\circ\text{C}$

$\Delta T [^\circ\text{C}]$	0	5	10	30	50	80	100	130	140
F_T [N]	0.00	0.14	0.27	0.82	1.36	2.18	2.72	3.54	3.81
ΔF_a [N]	6.15	5.94	5.72	4.85	3.98	2.68	1.81	0.51	0.07
$v_{a,L}$ [mm]	35.17	34.11	33.05	28.79	24.53	18.15	13.89	7.51	5.38
sT_L [mm]	0.00	1.06	2.12	6.38	10.64	17.02	21.28	27.66	29.79
$v_{a,NL}$ [mm]	31.84	30.99	30.13	26.62	23.03	17.43	13.55	7.48	5.38
sT_{NL} [mm]	0.00	0.85	1.71	5.22	8.81	14.41	18.29	24.36	26.46
h_L [mm]	103.07	102.01	100.95	96.69	92.43	86.05	81.79	67.90	73.28
h_{NL} [mm]	99.74	98.89	98.03	94.52	90.93	85.33	81.45	75.38	73.28
l_L [mm]	115.06	114.11	113.16	109.38	105.63	100.09	96.46	91.11	89.35
l_{NL} [mm]	112.09	111.32	110.56	107.47	104.32	99.48	96.17	91.08	89.35
Δl_L [mm]	30.06	29.11	28.16	24.38	20.63	15.09	11.46	6.09	4.36
Δl_{NL} [mm]	27.09	26.32	25.56	22.47	19.32	14.48	11.17	6.08	4.35
N_L [N]	4.17	4.02	3.86	3.26	2.66	1.77	1.19	0.34	0.55
N_{NL} [N]	3.60	3.47	3.35	2.86	2.36	1.60	1.09	0.32	0.05
$K_{a,L}$ [mm]	0.204	0.204	0.204	0.204	0.204	0.204	0.204	0.204	0.204
$K_{a,NL}$ [mm]	0.25	0.25	0.25	0.24	0.24	0.23	0.22	0.21	0.205
$\omega_{0,L}$ [rad/s]	16.70	16.96	17.23	18.46	19.99	23.25	26.57	36.14	42.70
$\omega_{0,NL}$ [rad/s]	17.55	17.79	18.04	19.19	20.64	23.72	26.91	36.21	42.70
$\Delta\omega_0$ [rad/s]	0.85	0.83	0.81	0.73	0.65	0.47	0.34	0.07	0.00

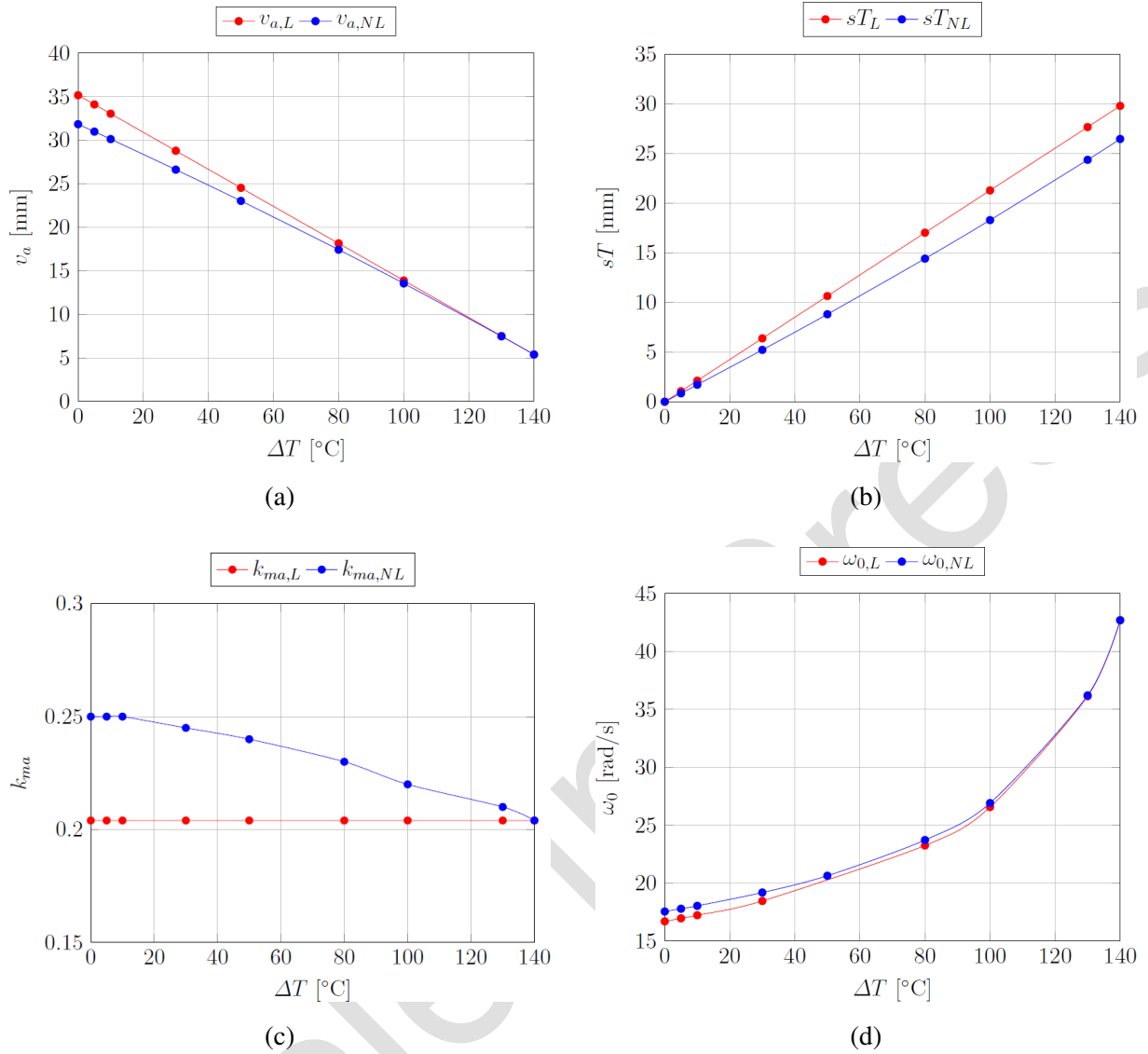


Fig. 9. Dependence of the calculated quantities of the V-actuator for Case 3, loaded with a force of $F_a = 10.07$ N ($m_a = 1.026$ kg) under various heating levels $\Delta T \in (0, 140)$ °C, and $k_T = 0.17$ mm/°C: (a) static deflection v_a , (b) thermal stroke sT , (c) stiffness k_{ma} , (d) natural angular frequency ω_0

large elastic deflection allows the springs to heat up to their limit values $\Delta T_{\max} = 140$ °C, thus achieving the larger thermal stroke of the actuator. The effect of spring heating significantly increases the natural frequency of the actuator. The current state of our measuring equipment did not allow verification of the calculated results for $\Delta T > 0$ °C by measurement. The measured values of the natural frequency of the actuator for the force $F_a = 10.07$ N and $\Delta T = 0$ °C are also presented in Table 1. Their agreement with the results of the nonlinear calculation is acceptable. To compare the magnitude of the thermal stroke, a calculation was made for an actuator with the span of $a = 0$ mm. For this parallel arrangement of the springs, $\Delta T_{\max} = 85$ °C and $sT_{NL} = 14.97$ mm. As can be seen from this comparison, the thermal stroke of the actuator with a spring span of 51.13 mm is 1.77 times larger. The thermal expansion coefficient is $k_T = 0.19$ mm/°C.

To assess the convergence of the geometrically nonlinear calculation of the thermoelastic deflection of the V-actuator, the results from Table 3 for a temperature rise of $\Delta T = 5$ °C

were selected. For $nmax = 1$ (linear solution), the static deflection equals 34.11 mm. The geometrically nonlinear deflection for $nmax = 100$ steps is 30.989 mm. For $nmax = 99$, the nonlinear static deflection is equal to 30.99 mm, for $nmax = 1\,000$ steps it is 30.968 mm, and for $nmax = 2\,000$ steps the deflection is 30.967 mm. With a decreasing number of load steps, the static deflection converges toward the linear solution obtained at $nmax = 1$. For example, for $nmax = 3$, the static deflection is 31.82 mm, and for $nmax = 2$, it is 32.30 mm. It follows that with an increasing number of steps, the static deflection solution converges nonlinearly from above and the computational model is stable. A similar convergence behaviour was observed for all other load cases. Therefore, the chosen number of 100 load steps is sufficient.

Case 4: *Elastostatic and thermoelastic deformation analyses and the natural angular frequency of the V-actuator loaded with a force of $F_a = 20\text{ N}$ ($m_a = 2.04\text{ kg}$) under various heating $\Delta T \in \langle 0, 140 \rangle^\circ\text{C}$*

In Case 4, the V-actuator, with the same parameters as presented in Case 1 is considered. Compared to Cases 2 and 3, the actuator is loaded with a larger force, $F_a = 20\text{ N}$ ($m_a = 2.04\text{ kg}$), and subjected to various heating $\Delta T \in \langle 0, 140 \rangle^\circ\text{C}$. The individual springs are then elongated by a force of $F_m = 12.52\text{ N}$. For this value of the force F_m , the coefficient of negative thermal expansion of the springs is $k_T = 0.058\text{ mm}/^\circ\text{C}$. The magnitude of this force is close to the maximum permissible force $F_{m,max} = 15.67\text{ N}$, for which $k_T = 0\text{ mm}/^\circ\text{C}$. The calculated results are shown in Table 4 and selected results are displayed in Fig. 10. The calculated quantities in Table 4 are labelled analogously as in Tables 2 and 3.

As the results in Table 4 and Fig. 10 show, the influence of geometric nonlinearity and

Table 4. Results of the elastostatic and thermoelastic deformation analyses and the natural frequencies for Case 4 of the V-actuator, loaded with a force of $F_a = 20\text{ N}$ ($m_a = 2.04\text{ kg}$) under various heating $\Delta T \in \langle 0, 140 \rangle^\circ\text{C}$ with $k_T = 0.058\text{ mm}/^\circ\text{C}$

$\Delta T [^\circ\text{C}]$	0	5	10	30	50	80	100	130	140
$v_{a,L} [\text{mm}]$	83.80	83.44	83.08	81.63	80.17	77.99	76.54	74.36	73.64
$sT_L [\text{mm}]$	0.00	0.36	0.72	2.17	3.63	5.81	7.26	9.44	10.16
$v_{a,NL} [\text{mm}]$	66.75	66.50	66.27	65.31	64.36	62.91	61.94	60.48	59.99
$sT_{NL} [\text{mm}]$	0.00	0.24	0.47	1.43	2.38	3.83	4.80	6.26	6.75
$h_L [\text{mm}]$	151.70	151.34	150.98	149.53	148.07	145.89	144.44	142.26	141.54
$h_{NL} [\text{mm}]$	134.64	134.41	134.17	133.21	132.36	130.81	129.85	128.38	127.89
$l_L [\text{mm}]$	160.09	159.745	159.40	158.03	156.65	154.59	153.23	151.17	150.49
$l_{NL} [\text{mm}]$	144.02	143.80	143.58	142.69	141.80	140.45	139.55	138.19	137.74
$\Delta l_L [\text{mm}]$	75.09	74.74	74.40	73.03	71.65	69.59	68.26	66.17	65.49
$\Delta l_{NL} [\text{mm}]$	59.03	58.80	58.58	57.69	56.80	55.45	54.55	53.19	52.74
$\varphi_L [^\circ]$	18.63	18.72	18.72	18.89	19.06	19.32	19.50	19.78	19.87
$\varphi_{NL} [^\circ]$	20.79	20.84	20.87	21.00	21.15	21.36	21.50	21.73	21.80
$N_L [\text{N}]$	11.37	11.32	11.26	11.04	10.82	10.49	10.27	9.47	9.84
$N_{NL} [\text{N}]$	9.03	8.99	8.95	8.79	8.63	8.39	8.23	7.99	7.91
$K_{a,L} [\text{N/mm}]$	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20
$K_{a,NL} [\text{N/mm}]$	0.31	0.31	0.31	0.31	0.31	0.31	0.30	0.30	0.30
$\omega_{0,L} [\text{rad/s}]$	10.82	10.84	10.87	10.96	11.06	11.21	11.32	11.48	11.54
$\omega_{0,NL} [\text{rad/s}]$	12.12	12.14	12.17	12.25	12.35	12.49	12.58	12.74	12.79
$\Delta\omega_0 [\text{rad/s}]$	1.30	1.30	1.30	1.29	1.29	1.28	1.28	1.26	1.25

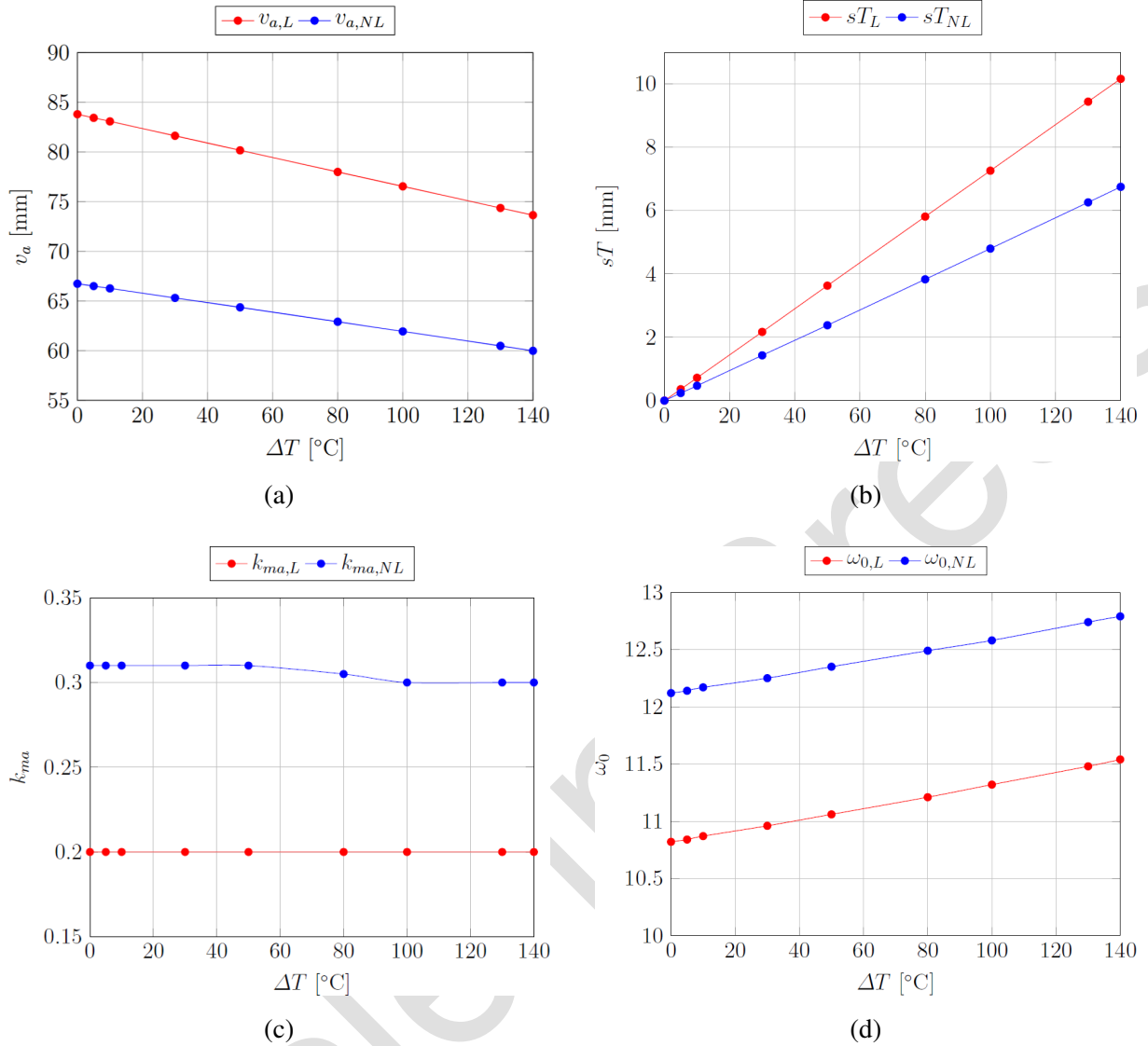


Fig. 10. Calculated quantities of the V-actuator as functions of temperature increases $\Delta T \in \langle 0, 140 \rangle$ °C for Case 4, under a constant force of $F_a = 20$ N ($m_a = 2.04$ kg) with $k_T = 0.058$ mm/°C: (a) static deflection v_a , (b) thermal stroke sT , (c) stiffness k_{ma} , (d) natural angular frequency ω_0

heating is more significant in comparison with the previous Cases 2 and 3. To compare the magnitude of the thermal stroke for $F_a = 20$ N, a calculation was made for an actuator with the span of $a = 0$ mm. The thermal expansion coefficient is equal to 0.10 mm/°C, which is almost twice as large as that for the spring span of 51.13 mm. For this parallel arrangement of the springs, $\Delta T_{\max} = 140$ °C and $sT_{NL} = 10.60$ mm. The thermal stroke of the actuator with the spring span of 51.13 mm is 1.54 times smaller, which is due to a significant reduction in the thermal expansion coefficient of the springs. The above results indicate that the actuation and resonant frequency of the V-actuator can be strongly dependent on the spring span. This effect will be investigated in our future research on this V-actuator system.

Table 5 presents a comparison of the values of thermal stroke, thermal expansion coefficient, static deflection and natural angular frequency at the ultimate heating levels of $\Delta T = 29$ °C for Case 2 and 30 °C for Cases 3 and 4 for a span of 51.13 mm. With an increasing value of the force F_a at approximately the same temperature rise, the thermal expansion coefficient

Table 5. Comparison of the thermal stroke, thermal expansion coefficient and natural angular frequency from Cases 2–5 for temperature rise of $\Delta T = 29^\circ\text{C}$ (Case 2), 30°C (Cases 3 and 4), and 0°C (Case 5)

Case	2	3	4	5
F_a [N]	5.55	10.07	20	30
F_m [N]	3.44	6.26	12.52	16.03
k_T [mm/ $^\circ\text{C}$]	0.22	0.17	0.058	0.00
sT_L [mm]	7.99	6.38	2.17	0.00
sT_{NL} [N]	7.71	5.22	1.43	0.00
$v_{a,L}$ [mm]	5.05	28.79	81.63	132.77
$v_{a,NL}$ [mm]	5.05	26.62	65.31	97.14
$\omega_{0,L}$ [rad/s]	44.07	18.46	10.95	9.52
$\omega_{0,NL}$ [rad/s]	44.07	19.99	12.25	10.05

and, consequently, the thermal stroke decrease. The increasing force F_a significantly increases the static deflection and significantly reduces the natural angular frequency. The significant influence of geometric nonlinearity on the static deflection is reflected in a less pronounced effect on the natural angular frequency. However, with an increasing force F_a , the heating can be increased up to its ultimate value to achieve a larger thermal stroke. The last column of Table 5 shows the results of the elastostatic deformation and modal analysis of the actuator loaded with a force of $F_a = 30\text{ N}$ (Case 5).

In Case 5, the individual springs of this actuator are loaded with a force $F_m = 16.03\text{ N}$. However, this force exceeds the permissible value of $F_{m,max} = 15.57\text{ N}$ for springs, at which the thermal expansion coefficient becomes zero. Consequently, the natural angular frequency for Case 5 is evaluated for the heating $\Delta T = 0^\circ\text{C}$ (elastostatic analysis).

Table 6 compares the static deflection, thermal stroke, and natural angular frequency for the permissible heating levels ΔT in Cases 2–4 at the corresponding applied force F_a . As expected for these ultimate states of the heating ΔT , the effect of geometric nonlinearity on the natural angular frequency is not significant, as the limiting heating minimizes the static deflection caused by the gravitational force F_a . However, an increasing force F_a significantly

Table 6. Comparison of the static deflection, thermal stroke, thermal expansion coefficient and natural angular frequency for Cases 2–4 at heating $\Delta T = 29^\circ\text{C}$ (Case2) and 140°C (Cases 3 and 4)

Case	2	3	4
$\Delta T_{\max}$ [$^\circ\text{C}$]	29	140	140
F_a [N]	5.55	10.07	20
F_m [N]	3.44	6.26	12.52
k_T [mm/ $^\circ\text{C}$]	0.22	0.17	0.058
sT_L [mm]	7.99	29.79	10.16
sT_{NL} [mm]	7.71	26.46	6.75
$v_{a,L}$ [mm]	5.05	5.38	73.64
$v_{a,NL}$ [mm]	5.05	5.38	59.99
$\omega_{0,L}$ [rad/s]	44.07	42.70	11.54
$\omega_{0,NL}$ [rad/s]	44.07	42.70	12.79

reduces the natural angular frequency. The smallest thermal stroke occurs in Case 4 with a force $F_a = 20$ N, which is caused by the smallest thermal expansion coefficient. Nevertheless, the weight lifted is almost double that of Case 3. The lowest natural angular frequency also corresponds to this larger load. In Case 4, the critical force ($F_{m,max} = 15.57$ N) has not yet been reached ($F_m = 12.52$ N), but further heating is no longer permissible due to the material properties of the nylon fiber.

4. Conclusions

Nylon springs with negative thermal expansion are used in engineering applications as low-cost and lightweight tensile thermomechanical actuators (artificial muscle). The tensile nylon springs under consideration exhibit geometric nonlinear stiffening and are characterized by an initial compressive prestress and negative thermal expansion. Due to this negative thermal expansion, heating a stretched spring loaded with an external mass generates a tensile thermomechanical force that lifts the mass (thermal stroke). The magnitude of the thermal stroke depends on the negative coefficient of thermal expansion and on the heating of the spring. In the case of spring elongation due to the weight of the attached mass, the maximum thermal stroke is limited by this elongation. However, the coefficient of thermal expansion significantly decreases with the increasing magnitude of the loading gravitational force. Consequently, the thermal stroke depends inversely on the magnitude of this gravitational force. Heating is also limited by a critical temperature at which the physical properties of the nylon fibre are adversely altered. Therefore, the application of a single spring for a high-tensile-force thermomechanical actuator has significant limitations. The actuation output of a single-spring actuator can be increased by various arrangements of multiple nylon springs.

This article deals with the geometrically nonlinear computational modelling and simulation of the static thermoelastic deflection of a tensile thermomechanical actuator formed by two nylon springs connected in a V-shape, referred to as the V-actuator. The external lumped mass m_a is connected at the junction node (actuation point) of the springs. This attached mass induces a tensile gravitational force F_a acting at the actuation point of the actuator.

The main motivation behind the V-actuator design is to increase its actuation stroke compared to a parallel arrangement of the same two springs. This study includes the calculation of the natural angular frequency of the undamped oscillation of the external mass attached to the V-actuator. This research also aims to determine how geometric nonlinearity and heating influence the static deformation, thermal stroke and natural frequency of the V-actuator carrying an external concentrated mass m_a .

Since the functionality of this actuator involves large changes in the initial length of the springs due to static gravitational and thermomechanical loading, the nonlinear theory of continuum mechanics and the finite element method are applied for their deformation analysis. Based on the updated Lagrange formulation of large displacements of a continuum, the incremental finite element equations are established for this actuator to calculate its static deflection caused by the attached concentrated mass and the heating of the springs.

Using this equilibrium static deflection, the natural frequency of the oscillating external mass is determined. The physical actuator, having a spring span of $a = 51.13$ mm, is modelled using custom nylon twisted coiled spring (NTCS) finite elements [22]. The established nonlinear equations are solved by the incremental Newton method considering the complex shape of the measured tensile characteristic of the physical nylon actuator prototype loaded with mechanical and thermomechanical forces. This measured tensile characteristics of the ac-

tuator is characterized by three regions. In the first region, the loading forces overcome the negative spring prestress. In the second, relatively short part of the characteristic, the actuator shows a linear dependence on the loading forces. In the third part of the tensile characteristic, the actuator exhibits nonlinear stiffening behaviour. The tensile characteristic is reversible and repeatable without significant hysteresis. The results of the performed numerical simulations of the V-actuator show a significant influence of geometric nonlinearity and heating on its static equilibrium deflection and the resonant frequency of the natural undamped vibration, especially during large elastostatic equilibrium elongations and thermomechanical shortenings of the springs.

The main contributions of this article consist of the creation of a physical and computational model of the thermomechanical tensile nylon V-actuator for calculating its static deflection and thermal stroke caused by mechanical and thermal loading, with a subsequent calculation of the undamped resonant frequency of the oscillating attached external mass. The established computational model includes the influence of geometrical nonlinearity and spring heating, as well as the initial response of the actuator to the non-conformal elongation of the springs when overcoming their initial compressive prestress. The magnitude of this elongation depends on the number of coils. For shorter springs, this part of the extension can be neglected. However, for longer springs, their inclusion in the computational model is essential. The results of the numerical calculations demonstrate the necessity of applying geometrically nonlinear models to the studied actuator, especially at large spring elongations. The influence of increased spring heating is extremely pronounced. It significantly reduces the static deflection and increases the natural frequency of the V-actuator. The selected numerical results achieved with the geometrically nonlinear model are in excellent agreement with experimental measurements performed on physical prototypes of the actuator. Another key contribution of this paper is demonstrating that a V-shaped arrangement of two springs multiplies the thermal stroke several-fold compared to a parallel alternative. However, there is a specific upper limit to the gravitational load F_a , attached to the actuator, beyond which a parallel spring arrangement becomes more efficient.

The algorithm for calculating the static deflection and the angular frequency of natural vibration of the studied actuators is processed using the software MATHEMATICA [35] into a custom code *Vnylonactuator*. The modelling and simulation procedure of the studied V-actuator can be applied to design other thermomechanical nylon actuators with different numbers and geometric arrangements of the tensile nylon springs.

Our future research will focus on expanding the capabilities for measuring and computational modelling of the thermoelastic and dynamic behaviour of physical actuators and dampers made of nylon springs. An important area of their mathematical and computational modelling is the optimization of their geometry, including alternative spring spans and multi-spring configurations and loading to meet the conditions for reliable functionality and efficiency. The effect of the spring span of the V-actuator on its thermal stroke and natural frequencies under different mechanical and thermal loads will be investigated. Furthermore, we will investigate the forced vibration of nylon actuators with both passive and active damping and the compiled mathematical models will be augmented to facilitate optimal automatic control of the physical electro-thermomechanical devices made of nylon springs.

Acknowledgement

The authors gratefully acknowledge financial support by the Slovak Research and Development Agency under Contract no. APVV-19-0406 and KEGA 006STU-4/2023.

References

- [1] Azis, S., Villacorta, B., Naficy, S., Salahuddin, B., Gao, S., Baigh, T. A., Sangian, D., Zhu, Z., Microwave powered polymeric artificial muscle, *Applied Materials Today* 23 (2021) No. 101021. <https://doi.org/10.1016/j.apmt.2021.101021>
- [2] Chen, J., Xu, M., Pakdel, E., Zhong, W., Li, X., Sun, L., Wang, D., Performance comparison of coiled made of PA6 single fiber and yarn, *Sensors and Actuators A: Physical* 346 (2022) No. 113822. <https://doi.org/10.1016/j.sna.2022.113822>
- [3] Chen, Y., Xie, Y., Liu, L., Liu, D., Effect of draw ratio on the action performance of twisted polymer artificial muscles, *Smart Materials and Structures* 34 (6) (2025) No. 065025. <https://doi.org/10.1088/1361-665X/ade2c3>
- [4] Cherubini, A., Moretti, G., Veretchny, R., Fontana, M., Experimental characterization of artificial thermally activated muscles based on coiled nylon fishing lines, *AIP Advances* 5 (6) (2015) No. 067158. <https://doi.org/10.1063/1.4923315>
- [5] Dai, J., Yuan, Y., Xu, H., Lin, S., Liu, C., Zhang, H., Lin, Z., Liang, Y., Ma, S., Woven coiled nylon artificial muscles with large tensile stress and high stability, *ACS Applied Polymer Materials* 7 (18) (2025) 12 187–12 196. <https://doi.org/10.1021/acsapm.5c01590>
- [6] Gálik, G., Sedlár, T., Goga, V., Uličný, M., Murín, J., Experimental measurement of nylon coiled spring properties for validation of numerical models, *AIP Conferences Proceedings* 3 054 (2024) No. 030003. <https://doi.org/10.1063/5.0187960>
- [7] Goga, V., Berta, Š., Murin, J., Šarkan, L., Universal device for measuring the stiffness of nylon springs and the method of measuring the stiffness of nylon tension springs, *Utility model PUV* 50042-2023, 2023.
- [8] Goga, V., Murín, J., Minár, M., Identification of dynamic parameters of the artificial muscles in form of the nylon springs, *Proceedings of the 14th International Conference on Vibration Problems (ICOVP 2019)*, 2019, pp. 307–315. https://doi.org/10.1007/978-981-15-8049-9_18
- [9] Haines, C. S., Lima, M. D., Li, N., Spinks, G. M., Foroughi, J., Madden, J. D. W., Kim, S. H., Fang, S. L., de Andrade, M. J., Göktepe, F., Göktepe, Ö., Mirvakili, S. M., Naficy, S., Lepró, X., Oh, J. Y., Kozlov, M. E., Kim, S. J., Xu, X. R., Swedlove, B. J., Wallace, G. G., Baughman, R. H., Artificial muscles from fishing line and sewing thread, *Science* 343 (6 173) (2014) 868–872. <https://doi.org/10.1126/science.1246906>
- [10] Huang, Y.-W., Li, W.-S., Yang, F., Lee, S., Tensile deformation of artificial muscles: Annealed nylon 6, *Polymer* 177 (2019) 49–56. <https://doi.org/10.1016/j.polymer.2019.05.070>
- [11] Jiang, Z., Huang, H., Huang, H., Ling, B., A truss type deployable manipulator actuated by parallel twisted and coiled nylon fiber actuator, *Proceedings of the 2020 IEEE International Conference on Real-time Computing and Robotics (RCAR)*, Asahikawa, Japan, 2020, pp. 105–110. <https://doi.org/10.1109/RCAR49640.2020.9303249>
- [12] Kikuta, T., Nishiwaki, T., Effect of twisted and coiled polymer actuator (TCPA) on crack dispersion properties of HPFRCC, *Materials* 15 (16) (2022) No. 5701. <https://doi.org/10.3390/ma15165701>
- [13] Li, T., Wand, Y., Liu, K., Liu, H., Zhang, J., Sheng, X., Guo, D., Thermal actuation performance modification of coiled artificial muscle by controlling annealing stress, *Journal of Polymer Science – Part B: Polymer Physics* 56 (5) (2018) 383–390. <https://doi.org/10.1002/polb.24551>
- [14] Liu, S., Tang, X., Zhou, D., Liu, Y., Fascicular module of nylon twisted actuators with large force and variable stiffness, *Sensors and Actuators A: Physical* 315 (2020) No. 112292. <https://doi.org/10.1016/j.sna.2020.112292>
- [15] Minár, M., Goga, V., Čapková, R., Ondřejčka, K., Murín, J., Basic parameters of coiled fishing line actuator, *Proceedings of the 2020 Cybernetics & Informatics (K&I)*, Velke Karlovice, Czech Republic, 2020, pp. 1–4. <https://doi.org/10.1109/KI48306.2020.9039870>

- [16] Murín, J., The formulation of a new nonlinear stiffness matrix of a finite element, *Computers & Structures* 54 (5) (1995) 933–938. [https://doi.org/10.1016/0045-7949\(94\)E0243-U](https://doi.org/10.1016/0045-7949(94)E0243-U)
- [17] Murín, J., Goga, V., Berta, Š., Šarkan, L., Universal device for measuring the stiffness of nylon springs and the method of measuring the stiffness of nylon tension springs, *Utility model PUV 50042-2023*, 2023.
- [18] Murín, J., Goga, V., Hrabovský, J., Paulech, J., Gálik, G., Kutiš, V., Influence of geometric non-linearity and heating on the dynamic parameters of free natural vibration of nylon springs with negative thermal expansion, *AIP Conference Proceedings* 3 455 (2026) No. 030001. <https://doi.org/10.1063/5.0327329>
- [19] Murín, J., Goga, V., Hrabovský, J., Paulech, J., Gálik, G., Šarkan, L., Kutiš, V., Structural analysis of new dampers made of nylon springs, *AIP Conference Proceedings* 2 778 (2023) No. 040019. <https://doi.org/10.1063/5.0135802>
- [20] Murín, J., Goga, V., Hrabovský, J., Paulech, J., Gálik, G., Šarkan, L., Kutiš, V., Aminbaghai, M., Nonlinear thermoelastic analysis of nylon springs with negative thermal expansion, *AIP Conference Proceedings* 3 054 (2024) No. 030009. <https://doi.org/10.1063/5.0187689>
- [21] Murín, J., Goga, V., Paulech, J., Gálik, G., Uličný, M., Kutiš, V., Influence of geometric nonlinearity and heating on the natural frequency of the V-actuator made of nylon springs, *Proceedings of the 40th conference Computational Mechanics (CM2025)*, University of West Bohemia in Pilsen, Sní, 2025, pp. 136–141.
- [22] Murín, J., Goga, V., Paulech, J., Hrabovský, J., Kutiš, V., Kugler, S., Aminbaghai, M., Geometric nonlinear elastic and thermoelastic analysis of the nylon springs with negative thermal expansion, *Sensors and Actuators A: Physical* 376 (2024) No. 115563. <https://doi.org/10.1016/j.sna.2024.115563>
- [23] Murín, J., Goga, V., Paulech, J., Hrabovský, J., Sedlár, T., Kutiš, V., Aminbaghai, M., Thermoelastic analysis of new dampers made of polymer springs with negative thermal expansion, *IOP Conference Series: Materials Science and Engineering* 1 199 (2021) No. 012056. <https://doi.org/10.1088/1757-899X/1199/1/012056>
- [24] Murín, J., Goga, V., Sedlár, T., Vibration damper with coiled tension springs made of nylon springs, *Utility model PUV 50026 – 2021*, 2021.
- [25] Murín, J., Paulech, J., Goga, V., Hrabovský, J., Kutiš, V., Kmotorka, R., Aminbaghai, M., Modeling and simulation of a torsion actuator made of nylon filament, *AIP Conference Proceedings* 2 411 (1) (2021) No. 090003. <https://doi.org/10.1063/5.0067641>
- [26] Nishimura, Y., Spinks, G. M., Detailing the visco-elastic origin of thermo-mechanical training of twisted and coiled polymer fiber artificial muscle, *Jornal of Polymer Science* 60 (8) (2022) 1 360–1 370. <https://doi.org/10.1002/pol.20220024>
- [27] Novák, P., Búc, D., Murín, J., Goga, V., Analysis of twisted polymeric fibers by X-ray diffraction, *AIP Conference Proceedings* 2 411 (2021) No. 040009. <https://doi.org/10.1063/5.0067338>
- [28] Piao, C., Jang, H., Lim, T. G., Kim, H., Choi, H. R., Hao, Y., Suk, J. W., Enhanced dynamic performance of twisted and coiled soft actuators using graphene coating, *Composite Part B: Engineering* 178 (2019) No. 107499. <https://doi.org/10.1016/j.compositesb.2019.107499>
- [29] Schenk, N., Benecke, L., Chokri, C., Review of thermally activated twisted coiled polymer actuators: A materials science perspective, *Advanced Functional Materials* 36 (58) (2026) No. e76552. <https://doi.org/10.1002/adfm.76552>
- [30] Stanojevic, N., Chauhan, S., Strain and temperature sensors made from fishing lines, *Engineering Research Express* 4 (3) (2022) No. 035036. <https://doi.org/10.1088/2631-8695/ac8913>
- [31] Stanojevic, N., Zhang, J., Chauhan, S., Numerical modelling and experimental study of a super-coiled polymer for dual actuator-sensor applications, *Sensors and Actuators A: Physical* 361 (2023) No. 114598. <https://doi.org/10.1016/j.sna.2023.114598>

- [32] Sun, J., Zhang, S., Deng, J., Li, J., Wang, D., Liu, J., Liu, Y., Recent advances in twisted coiled artificial muscles and their applications, *SmartBot* 1 (1) (2025) No. e12005.
<https://doi.org/10.1002/smb2.12005>
- [33] Suzuki, M., Kamamichi, N., Displacement control of antagonistic type twisted and coiled polymer actuator, *Smart Materials and Structures* 27 (3) (2018) No. 035003.
<https://doi.org/10.1088/1361-665X/aaa587>
- [34] Takagi, K., Iwai, H., Oiwa, C., Irisawa, T., Shioya, M., Masuya, K., Tahara, K., Sakurai, D., Watanabe, H., Asaka, K., Modelling and characterization for straight twisted polymer fiber actuators in blocked torsion: Effect of radial thermal expansion, *Smart Materials and Structures* 30 (6) (2021) No. 065023. <https://doi.org/10.1088/1361-665X/abfc1a>
- [35] Wolfram MATHEMATICA 9.0.1.0. Wolfram Research, 2013.
- [36] Wu, C., Zheng, W., A modelling of twisted and coiled polymer artificial muscles based on elastic theory, *Actuators* 9 (2) (2020) No. 25. <https://doi.org/10.3390/act9020025>
- [37] Yang, Q., Li, G., A top-down multi-scale modeling for actuation response of polymeric artificial muscles, *Journal of the Mechanics and Physics of Solids* 92 (2016) 237–259.
<https://doi.org/10.1016/j.jmps.2016.04.007>
- [38] Yun, S., Mun, S., Park, S. K., Hwang, I., Choi, M., Thermo-mechanically robust compliant electrode based on surface modification of twisted and coiled nylon-6 fiber for artificial muscle with highly durable contractile stroke, *Polymers* 14 (17) (2022) No. 3601.
<https://doi.org/10.3390/polym14173601>
- [39] Zhang, J., Ply for a large stroke, *Sciences* 387 (6 738) (2025) 1 038-1 039.
<https://doi.org/10.1126/science.adw2181>