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Dynamic response of nuclear fuel assembly excited by pressure pulsations

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Abstract

The paper deals with dynamic load calculation of the hexagonal type nuclear fuel assembly caused by spatial motion of the support plates in the reactor core. The support plate motion is excited by pressure pulsations generated by main circulation pumps in the coolant loops of the primary circuit of the nuclear power plant. Slightly different pumps revolutions generate the beat vibrations which causes an amplification of fuel assembly component dynamic deformations and fuel rods coating abrasion. The cyclic and central symmetry of the fuel assembly makes it possible the system decomposition into six identical revolved fuel rod segments which are linked with central tube and skeleton by several spacer grids in horizontal planes.

The modal synthesis method with condensation of the fuel rod segments is used for calculation of the normal and friction forces transmitted between fuel rods and spacer grids cells.

Keywords: vibrations, nuclear fuel assembly, pressure pulsations, modal synthesis method, dynamic load

1. Introduction

An assessment of nuclear fuel assemblies (FA) behaviour at standard operating conditions of the nuclear reactors belongs to important safety and reliability audits. A significant part of FA assessment plays dynamic deformations and load of FA components and abrasion of fuel rods coating [6]. Dynamic properties of FA are usually investigated experimentally using their physical models [4, 7]. Frequency lowest modal values (eigenfrequencies and eigenvectors) investigated by measurement in the air, serve as initial data for parametric identification of the FA global model of beam type [9] used in dynamic analyses of the VVER1000 type reactors [2, 11]. These FA global models do not enable investigation of dynamic deformations of FA components and dynamic coupling forces between FA components taken to fuel road coating assessment.

The goal of this paper, in direct sequence at an interpretation of FA modelling and FA modal analysis published in the paper [13], is a presentation of the newly developed method for calculation of the hexagonal type FA component dynamic load caused by pressure pulsations generated by main circulation pumps in coolant loops [5] of the primary circuit of the nuclear power plants (NPP). The method is applied to Russian TVSA-T FA [1, 8] installed in reactor VVER1000 in NPP Temelín.

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2. Mathematical model of the FA kinematical excited vibration

In order to model the hexagonal FA can be divided into six rod segments (S), centre tube (CT) and load-bearing skeleton (LS) (see Fig. 1). Each rod segment (on Fig. 2 drawn in lateral cross-section and circumscribed by triangles) is composed of many fuel rods with fixed bottom ends in the lower tailpiece and several more guide thimbles fully restrained in lower (LP) and head pieces (HP). The centre tube is fully restrained into these pieces. The skeleton is created of six angle pieces (AP) fast linked with LP and mutually coupled by divided grid rims (GR). All FA components are linked by transverse spacer grids of three types (SG_1–SG_3) which elastic properties are expressed by linear springs placed on several level spacings $g = 1, \dots, G$ (see Fig. 1). The fuel rods are embedded into spacer grids with small initial tension, which would not fall below zero during core operation.

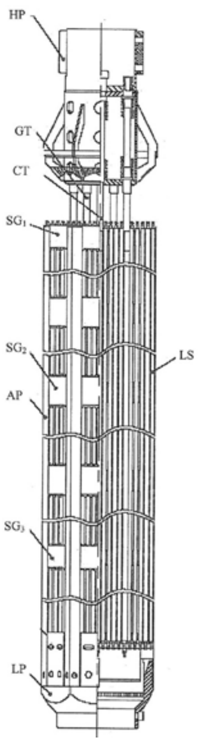

Fig. 1. Scheme of the fuel assembly

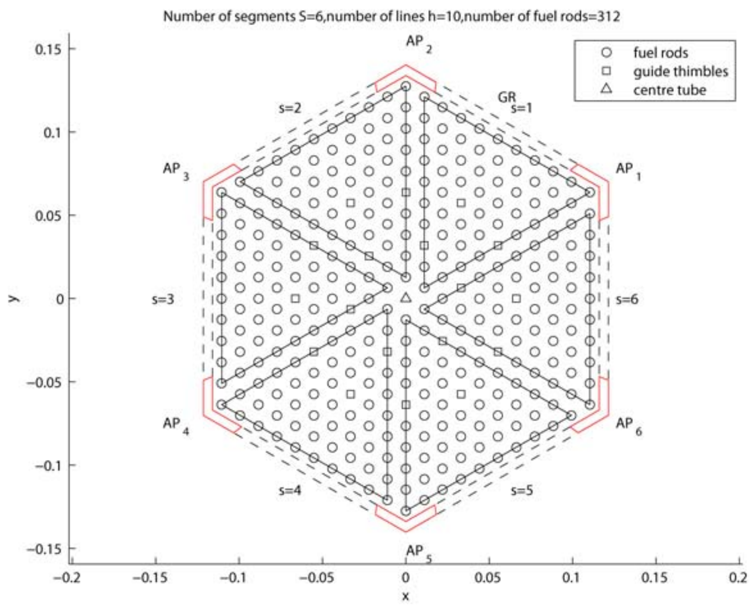

Fig. 2. The FA cross-section

Each FA is fixed by means of lower tailpiece (LP) into mounting plate in core barrel bottom and by means of head piece (HP) into lower supporting plate of the block of protection tubes. These support plates with pieces can be considered in transverse direction as rigid bodies.

Let use consider the spatial motion of the support plates described in coordinate systems x_X, y_X, z_X ($X = L, U$) with origins in plate gravity centres L, U by displacement vectors (see Fig. 3)

$$\boldsymbol{q}_X = [x_X, y_X, z_X, \varphi_{x,X}, \varphi_{y,X}, \varphi_{z,X}]^T, \quad X = L, U\,. \tag{1}$$

The lateral $\xi^{(s)}_{r,X}$, $\eta^{(s)}_{r,X}$ and bending $\vartheta^{(s)}_{r,X}$, $\psi^{(s)}_{r,X}$ displacements in the end-nodes of the fuel rod or guide thimbles r in segment s (in Fig. 3 illustrated for $s = 1$) coupled with plates can be expressed by the displacements of the lower ($X = L$) and upper ($X = U$) plates in the form

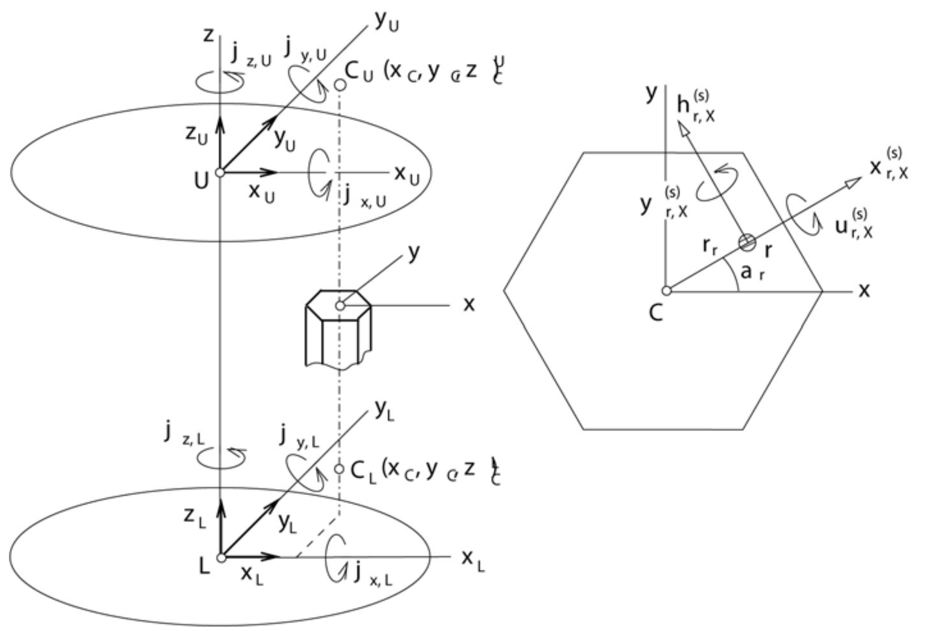

Fig. 3. Spatial motion of the FA support plates

$$\begin{bmatrix} \xi_{r,X}^{(s)} \\ \eta_{r,X}^{(s)} \\ \vartheta_{r,X}^{(s)} \\ \psi_{r,X}^{(s)} \end{bmatrix} = \begin{bmatrix} C_r^{(s)} & S_r^{(s)} & 0 & -z_C^X S_r^{(s)} & z_C^X C_r^{(s)} & x_C S_r^{(s)} - y_C C_r^{(s)} \\ -S_r^{(s)} & C_r^{(s)} & 0 & -z_C^X C_r^{(s)} & -z_C^X S_r^{(s)} & x_C C_r^{(s)} + y_C S_r^{(s)} + r_r \\ 0 & 0 & 0 & C_r^{(s)} & S_r^{(s)} & 0 \\ 0 & 0 & 0 & -S_r^{(s)} & C_r^{(s)} & 0 \end{bmatrix} \begin{bmatrix} x_X \\ y_X \\ z_X \\ \varphi_{x,X} \\ \varphi_{y,X} \\ \varphi_{z,X} \end{bmatrix}, \tag{2}$$

shortly

$$\boldsymbol{q}_{r,X}^{(s)} = \boldsymbol{T}_{r,X}^{(s)} \boldsymbol{q}_X, \quad X = L, U\,, \tag{3}$$

where x_C, y_C, z_C^X, $X = L, U$ are coordinates of the FA lower C_L and upper C_U piece centres in the coordinate systems of the support plates. Values $C_r^{(s)}$ and $S_r^{(s)}$ corresponding to $r-$th fuel rod (guide thimble) in segment s for the hexagonal type FA are

$$C_r^{(s)} = \cos\left[\alpha_r + (s-1)\frac{\pi}{3}\right], \quad S_r^{(s)} = \sin\left[\alpha_r + (s-1)\frac{\pi}{3}\right] \tag{4}$$

and r_r, α_r are polar coordinates of the fuel rod (guide thimble) centre in transverse plane (Fig. 3). The total transformations between displacements of the all kinematical excited nods of the subsystem components and lower (L) or upper (U) support plate displacements can be expressed according to (2) and (3) in the global matrix form

$$\boldsymbol{q}_L^{(s)} = \boldsymbol{T}_L^{(s)} \boldsymbol{q}_L,\; s = 1, \dots, 6, CT, LS\,; \quad \boldsymbol{q}_U^{(s)} = \boldsymbol{T}_U^{(s)} \boldsymbol{q}_U,\; s = 1, \dots, 6, CT\,. \tag{5}$$

The Russian TVSA-T FA (Fig. 1) in NPP Temelín contains in each segment $s \in \{1, \dots, 6\}$ 52 fuel rods and 3 guide thimbles at the positions 5, 20, 30. Therefore the transformation relations (5) for these subsystems have the form

$$\begin{bmatrix} \boldsymbol{q}_{1,L}^{(s)} \\ \vdots \\ \boldsymbol{q}_{r,L}^{(s)} \\ \vdots \\ \boldsymbol{q}_{55,L}^{(s)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{T}_{1,L}^{(s)} \\ \vdots \\ \boldsymbol{T}_{r,L}^{(s)} \\ \vdots \\ \boldsymbol{T}_{55,L}^{(s)} \end{bmatrix} \boldsymbol{q}_L \,, \qquad \begin{bmatrix} \boldsymbol{q}_{5,U}^{(s)} \\ \boldsymbol{q}_{20,U}^{(s)} \\ \boldsymbol{q}_{30,U}^{(s)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{T}_{5,U}^{(s)} \\ \boldsymbol{T}_{20,U}^{(s)} \\ \boldsymbol{T}_{30,U}^{(s)} \end{bmatrix} \boldsymbol{q}_U \tag{6}$$

and transformation matrices are of type $\boldsymbol{T}_L^{(s)} \in R^{220,6}$ and $\boldsymbol{T}_U^{(s)} \in R^{12,6}$.

The vectors of generalized coordinates of the fully restrained subsystems (rod segments and centre tube) loosed in kinematical excited nodes can be partitioned in the form

$$\boldsymbol{q}_s = [(\boldsymbol{q}_L^{(s)})^T, (\boldsymbol{q}_F^{(s)})^T, (\boldsymbol{q}_U^{(s)})^T]^T, \; s = 1, \ldots, 6, CT \tag{7}$$

and the skeleton $s = LS$ fixed only in bottom ends in the form

$$\boldsymbol{q}_{LS} = [(\boldsymbol{q}_L^{(LS)})^T, (\boldsymbol{q}_F^{(LS)})^T]^T \,. \tag{8}$$

The displacements of free system nodes (uncoupled with support plates) are integrated in vectors $\boldsymbol{q}_F^{(s)} \in R^{n_s}$. The conservative mathematical models of the loosed subsystems in the decomposed block form corresponding to partitioned vectors can be written as

$$\begin{bmatrix} \boldsymbol{M}_L^{(s)} & \boldsymbol{M}_{L,F}^{(s)} & \boldsymbol{0} \\ \boldsymbol{M}_{F,L}^{(s)} & \boldsymbol{M}_F^{(s)} & \boldsymbol{M}_{F,U}^{(s)} \\ \boldsymbol{0} & \boldsymbol{M}_{U,F}^{(s)} & \boldsymbol{M}_U^{(s)} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{q}}_L^{(s)} \\ \ddot{\boldsymbol{q}}_F^{(s)} \\ \ddot{\boldsymbol{q}}_U^{(s)} \end{bmatrix} + \begin{bmatrix} \boldsymbol{K}_L^{(s)} & \boldsymbol{K}_{L,F}^{(s)} & \boldsymbol{0} \\ \boldsymbol{K}_{F,L}^{(s)} & \boldsymbol{K}_F^{(s)} & \boldsymbol{K}_{F,U}^{(s)} \\ \boldsymbol{0} & \boldsymbol{K}_{U,F}^{(s)} & \boldsymbol{K}_U^{(s)} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_L^{(s)} \\ \boldsymbol{q}_F^{(s)} \\ \boldsymbol{q}_U^{(s)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_L^{(s)} \\ \boldsymbol{f}_C^{(s)} \\ \boldsymbol{f}_U^{(s)} \end{bmatrix} \tag{9}$$

for the $s = 1, \ldots, 6, CT$ and for the skeleton as

$$\begin{bmatrix} \boldsymbol{M}_L^{(LS)} & \boldsymbol{M}_{L,F}^{(LS)} \\ \boldsymbol{M}_{F,L}^{(LS)} & \boldsymbol{M}_F^{(LS)} \end{bmatrix} \begin{bmatrix} \ddot{\boldsymbol{q}}_L^{(LS)} \\ \ddot{\boldsymbol{q}}_F^{(LS)} \end{bmatrix} + \begin{bmatrix} \boldsymbol{K}_L^{(LS)} & \boldsymbol{K}_{L,F}^{(LS)} \\ \boldsymbol{K}_{F,L}^{(LS)} & \boldsymbol{K}_F^{(LS)} \end{bmatrix} \begin{bmatrix} \boldsymbol{q}_L^{(LS)} \\ \boldsymbol{q}_F^{(LS)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_L^{(LS)} \\ \boldsymbol{f}_C^{(LS)} \end{bmatrix}, \tag{10}$$

where letters $\boldsymbol{M}$ $(\boldsymbol{K})$ correspond to mass (stiffness) submatrices of the subsystems. The force subvectors $\boldsymbol{f}_C^{(s)}$ express the coupling forces between subsystem s and adjacent subsystems transmitted by spacer grids. The second set of equations extracted from (9) and (10) for each subsystem is

$$\begin{aligned} \boldsymbol{M}_F^{(s)} \ddot{\boldsymbol{q}}_F^{(s)} + \boldsymbol{K}_F^{(s)} \boldsymbol{q}_F^{(s)} \;=\; & -\boldsymbol{M}_{F,L}^{(s)} \boldsymbol{T}_L^{(s)} \ddot{\boldsymbol{q}}_L - \boldsymbol{M}_{F,U}^{(s)} \boldsymbol{T}_U^{(s)} \ddot{\boldsymbol{q}}_U - \\ & \boldsymbol{K}_{F,L}^{(s)} \boldsymbol{T}_L^{(s)} \boldsymbol{q}_L - \boldsymbol{K}_{F,U}^{(s)} \boldsymbol{T}_U^{(s)} \boldsymbol{q}_U + \boldsymbol{f}_C^{(s)} \,, \end{aligned} \tag{11}$$

where $\boldsymbol{M}_{F,U}^{(LS)} = \boldsymbol{0}$, $\boldsymbol{K}_{F,U}^{(LS)} = \boldsymbol{0}$ because the skeleton (LS) is fixed only with lower support plate.

The global model of the FA has to large DOF number for calculation of dynamic response excited by support plate motion. Therefore we assemble the condensed model using the modal synthesis method presented in the paper [12]. Let the modal properties of the conservative models of the mutually uncoupled subsystems with the strengthened end-nodes coupled with immovable support plates be characterized by spectral $\boldsymbol{\Lambda}_s$ and modal $\boldsymbol{V}_s$ matrices of order n_s, suitable to orthonormality conditions

$$\boldsymbol{V}_s^T \boldsymbol{M}_F^{(s)} \boldsymbol{V}_s = \boldsymbol{E} \,, \quad \boldsymbol{V}_s^T \boldsymbol{K}_F^{(s)} \boldsymbol{V}_s = \boldsymbol{\Lambda}_s, \quad s = 1, \ldots, 6, CT, LS \,. \tag{12}$$

The vectors $\boldsymbol{q}_F^{(s)}$ of dimension n_s, corresponding to free nodes of subsystems, can be approximately transformed in the form

$$\boldsymbol{q}_F^{(s)} = {}^m\boldsymbol{V}_s \boldsymbol{x}_s \,, \quad \boldsymbol{x}_s \in R^{m_s} \,, \quad s = 1, \ldots, 6, CT, LS \,, \tag{13}$$

where ${}^m\boldsymbol{V}_s \in R^{n_s,m_s}$ are modal submatrices compound out of chosen m_s master eigenvectors of fixed subsystems. The equations (11) can be rewritten using (12) and (13) in the form

$$\begin{aligned} \ddot{\boldsymbol{x}}_s + {}^m\boldsymbol{\Lambda}_s \boldsymbol{x}_s \;=\; & -{}^m\boldsymbol{V}_s^T (\boldsymbol{M}_{F,L}^{(s)} \boldsymbol{T}_L^{(s)} \ddot{\boldsymbol{q}}_L + \boldsymbol{M}_{F,U}^{(s)} \boldsymbol{T}_U^{(s)} \ddot{\boldsymbol{q}}_U + \boldsymbol{K}_{F,L}^{(s)} \boldsymbol{T}_L^{(s)} \boldsymbol{q}_L + \\ & \boldsymbol{K}_{F,U}^{(s)} \boldsymbol{T}_U^{(s)} \boldsymbol{q}_U) + {}^m\boldsymbol{V}_s^T \boldsymbol{f}_C^{(s)} \,, \qquad s = 1, \ldots, 6, CT, LS \,, \end{aligned} \tag{14}$$

where spectral submatrices ${}^m\boldsymbol{\Lambda}_s \in R^{m_s,m_s}$ correspond to chosen master eigenvectors in ${}^m\boldsymbol{V}_s$. The models (14) of all subsystems can be written in the configuration space $\boldsymbol{x} = [\boldsymbol{x}_s]$, $s = 1, \ldots, 6, CT, LS$ of dimension $m = \sum_s m_s$ as

$$\ddot{\boldsymbol{x}}(t) + \boldsymbol{\Lambda} \boldsymbol{x}(t) = -\boldsymbol{V}^T (\boldsymbol{M}_L \ddot{\boldsymbol{Q}}_L + \boldsymbol{M}_U \ddot{\boldsymbol{Q}}_U + \boldsymbol{K}_L \boldsymbol{Q}_L + \boldsymbol{K}_U \boldsymbol{Q}_U) + \boldsymbol{V}^T \boldsymbol{f}_C \,, \tag{15}$$

where $\boldsymbol{f}_C = [\boldsymbol{f}_C^{(s)}] \in R^n$, $n = \sum_s n_s$ is global vector of coupling forces between subsystems and matrices

$$\begin{aligned} \boldsymbol{\Lambda} \;&=\; \mathrm{diag}[{}^m\boldsymbol{\Lambda}_s] \in R^{m,m}; \quad \boldsymbol{V} = \mathrm{diag}[{}^m\boldsymbol{V}_s] \in R^{n,m};\; \boldsymbol{X}_X = \mathrm{diag}[\boldsymbol{X}_{F,X}^{(s)} \boldsymbol{T}_X^{(s)}] \in R^{n,48} \\ \boldsymbol{X} \;&=\; \boldsymbol{M}, \boldsymbol{K}; \quad X = L, U;\; s = 1, \ldots, 6, CT, LS \end{aligned}$$

are block diagonal, composed from corresponding matrices of subsystems. Vectors $\boldsymbol{Q}_X = [\boldsymbol{q}_X^T, \ldots, \boldsymbol{q}_X^T]^T \in R^{48}$, $X = L, U$ are assembled for eight FA subsystems from eight times repeating support plate displacement vectors. The global vector of coupling forces between subsystems can be calculated from identity

$$\boldsymbol{f}_C = -\frac{\partial E_p}{\partial \boldsymbol{q}_F} = -\boldsymbol{K}_C \boldsymbol{q}_F, \quad \boldsymbol{q}_F = [\boldsymbol{q}_F^{(s)}] \,, \tag{16}$$

where E_p is potential (deformation) energy of the all spacer grids (springs) between subsystems. The stiffness matrix $\boldsymbol{K}_C$ of all couplings between subsystems was derived in [13] for Russian TVSA-T fuel assembly. The expressions (16) can be substituted in (15) and then we get the condensed model of the nuclear fuel assembly of order m

$$\ddot{\boldsymbol{x}}(t) + (\boldsymbol{\Lambda} + \boldsymbol{V}^T \boldsymbol{K}_C \boldsymbol{V}) \boldsymbol{x}(t) = -\boldsymbol{V}^T (\boldsymbol{M}_L \ddot{\boldsymbol{Q}}_L(t) + \boldsymbol{M}_U \ddot{\boldsymbol{Q}}_U(t) + \boldsymbol{K}_L \boldsymbol{Q}_L(t) + \boldsymbol{K}_U \boldsymbol{Q}_U(t)) \,. \tag{17}$$

3. Fuel assembly steady vibration and fuel rod coating abrasion caused by pressure pulsations

The steady vibrations of the reactor VVER1000 excited by coolant pressure pulsations in the gap between core barrel and reactor pressure vessel walls generated by the main circulation pumps were investigated in co-operation with NRI Řež [10] and published in [11]. The force effect can be expressed in the global model of the reactor by excitation vector in the complex form [11]

$$\boldsymbol{f}(t) = \sum_j \sum_k \boldsymbol{f}_j^{(k)} \mathrm{e}^{\mathrm{i} k \omega_j t} \,, \tag{18}$$

where $\boldsymbol{f}_j^{(k)}$ is vector of complex amplitudes of $k-$th excitation harmonic component caused by hydrodynamic forces generated in one $j-$th circulation pump. Corresponding angular rotational frequency of the $j-$th pump $\omega_j = 2\pi f_j$ is defined by pump revolutions per minute n_j [rpm], where can be for particular pumps slightly different. Steady dynamic response of the reactor in generalized coordinates is given by identical form [11]

$$\boldsymbol{q}(t) = \sum_j \sum_k \boldsymbol{q}_j^{(k)} \mathrm{e}^{\mathrm{i}k\omega_j t} . \tag{19}$$

The vectors of complex amplitudes $\boldsymbol{q}_j^{(k)}$ must be transformed into vectors of $\boldsymbol{Q}_{X,j}^{(k)}$ $(X = L, U)$ describing steady vibration of the support plates caused by $k-$th harmonic of $j-$th pump [13, 15].

In consequence of slightly damped fuel assembly components we consider modal damping of the subsystems characterized in the space of modal coordinates $\boldsymbol{x}_s$ by diagonal matrices $\boldsymbol{D}_s = \mathrm{diag}[2D_\nu^{(s)}\Omega_\nu^{(s)}]$, where $D_\nu^{(s)}$ are damping factors of natural modes and $\Omega_\nu^{(s)}$ are eigenfrequencies of the mutually uncoupled subsystems. The damping of spacer grids can be approximately expressed by damping matrix $\boldsymbol{B}_C = \beta \boldsymbol{K}_C$ proportional to stiffness matrix $\boldsymbol{K}_C$ by coefficient β.

That being simplifying supposed and the polyharmonic excitation (18) the conservative condensed model (17) will be completed in the complex form

$$\begin{aligned}&\ddot{\boldsymbol{x}}(t) + (\boldsymbol{D} + \beta \boldsymbol{V}^T \boldsymbol{K}_C \boldsymbol{V})\dot{\boldsymbol{x}}(t) + (\boldsymbol{\Lambda} + \boldsymbol{V}^T \boldsymbol{K}_C \boldsymbol{V})\boldsymbol{x}(t) = \\ &= -\boldsymbol{V}^T \sum_j \sum_k \left[(\boldsymbol{K}_L - k^2\omega_j^2 \boldsymbol{M}_L)\boldsymbol{Q}_{L,j}^{(k)} + (\boldsymbol{K}_U - k^2\omega_j^2 \boldsymbol{M}_U)\boldsymbol{Q}_{U,j}^{(k)} \right] \mathrm{e}^{\mathrm{i}k\omega_j t} .\end{aligned} \tag{20}$$

Steady response of the fuel assembly subsystems in the complex form according to (13) is

$$\widetilde{\boldsymbol{q}}_F^{(s)}(t) = \sum_j \sum_k {}^m\boldsymbol{V}_s \widetilde{\boldsymbol{x}}_{s,j}^{(k)} \mathrm{e}^{\mathrm{i}k\omega_j t} , \quad s = 1, \dots, 6, CT, LS , \tag{21}$$

where $\widetilde{\boldsymbol{x}}_{s,j}^{(k)}$ are subvectors of the global vector $\widetilde{\boldsymbol{x}}_j^{(k)}$ of the complex amplitudes

$$\begin{aligned}\widetilde{\boldsymbol{x}}_j^{(k)} \;&=\; -[\boldsymbol{\Lambda} + (1 + \mathrm{i}\beta k\omega_j)\boldsymbol{V}^T \boldsymbol{K}_C \boldsymbol{V} + \mathrm{i}k\omega_j \boldsymbol{D}]^{-1} \cdot \\ &\quad \boldsymbol{V}^T \sum_j \sum_k \left[(\boldsymbol{K}_L - k^2\omega_j^2 \boldsymbol{M}_L)\boldsymbol{Q}_{L,j}^{(k)} + (\boldsymbol{K}_U - k^2\omega_j^2 \boldsymbol{M}_U)\boldsymbol{Q}_{U,j}^{(k)} \right]\end{aligned} \tag{22}$$

corresponding to subsystem s. Subscript $j \in \{1, 2, 3, 4\}$ is assigned to the operating circulation pump and subscript k to the harmonic component of pressure pulsations. The real steady dynamic response expressed by the generalized coordinates vector of the FA subsystems s in dependence on time according to (21) and (22) is

$$\boldsymbol{q}_F^{(s)}(t) = \sum_j \sum_k {}^m\boldsymbol{V}_s \left(\mathrm{Re}[\widetilde{\boldsymbol{x}}_{s,j}^{(k)}] \cos k\omega_j t - \mathrm{Im}[\widetilde{\boldsymbol{x}}_{s,j}^{(k)}] \sin k\omega_j t \right) . \tag{23}$$

The components of of the vector generalized coordinates $\boldsymbol{q}_F^{(s)}(t)$ corresponding to rod segment s are nodal points displacements of particular fuel rod or guide thimble on the level all spacer grids g [13] in the form

$$\boldsymbol{q}_F^{(s)} = [\dots, \xi_{r,g}^{(s)}, \eta_{r,g}^{(s)}, \vartheta_{r,g}^{(s)}, \psi_{r,g}^{(s)}, \dots] , \quad r = 1, \dots, R; \quad g = 1, \dots, G , \tag{24}$$

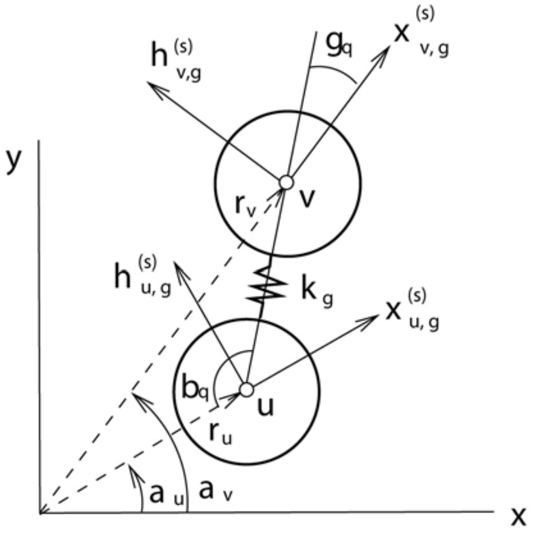

Fig. 4. The coupling between two fuel rods

where R is number of fuel rods and guide thimbles in one rod segment $s \in \{1, \ldots, 6\}$ and G is number of spacer grids.

The dynamic normal force transmitted by spacer grid between two adjacent fuel rods u and v (see Fig. 4) of the segment s on the level spacer grid g can be expressed in the form

$$N_{q,g}^{(s)} = -k_g \left[\xi_{v,g}^{(s)} \cos \gamma_q + \eta_{v,g}^{(s)} \sin \gamma_q + \xi_{u,g}^{(s)} \cos \beta_q - \eta_{u,g}^{(s)} \sin \beta_q\right] , \tag{25}$$

where k_g is stiffness of the transverse spring expressing the spacer grid cell stiffness between two adjacent fuel rods. Angles β_q, γ_q correspond to fuel rod couple u and v that is assigned coupling q. The fuel rod positions in the rod segment are determined by polar coordinates r_u, α_u and r_v, α_v of the linked fuel rods [3]. The slip speeds between transverse vibrating spacer grid on the level g and bending vibrating fuel rods u and v due to fuel rods bending inside of spacer grid cell are

$$c_{u,g}^{(s)} = r(\sin \beta_q \dot{\vartheta}_{u,g}^{(s)} + \cos \beta_q \dot{\psi}_{u,g}^{(s)}) , \quad c_{v,g}^{(s)} = r(-\sin \gamma_q \dot{\vartheta}_{v,g}^{(s)} + \cos \gamma_q \dot{\psi}_{v,g}^{(s)}) , \tag{26}$$

where r is outside diameter of the fuel rod coating. Bending angular velocities of fuel rod cross-section are expressed by corresponding components of the vector

$$\dot{\boldsymbol{q}}_F^{(s)}(t) = -\sum_j \sum_k k\omega_j \boldsymbol{V}_s \left(\mathrm{Re}[\widetilde{\boldsymbol{x}}_{s,j}^{(k)}] \sin k\omega_j t + \mathrm{Im}[\widetilde{\boldsymbol{x}}_{s,j}^{(k)}] \cos k\omega_j t\right) \tag{27}$$

obtained by the derivative of generalized coordinate vector (23) with respect to time. The power of the friction forces in the contact of the fuel rod coating and spacer grid cell is

$$P_{u,g}^{(s)} = f N_{q,g}^{(s)} c_{u,g}^{(s)} \text{ and } P_{v,g}^{(s)} = f N_{q,g}^{(s)} c_{v,g}^{(s)} , \tag{28}$$

where f is friction coefficient. The criterion of the fuel rod coating abrasion can be expressed by the work of the friction forces during the period T of the first harmonic component of pressure pulsations

$$W_{u,g}^{(s)} = \int_0^T |P_{u,g}^{(s)}| \, \mathrm{d}t \text{ or } W_{v,g}^{(s)} = \int_0^T |P_{v,g}^{(s)}| \, \mathrm{d}t \tag{29}$$

from the moment of maximal dynamic force transmitted by extreme stressed spacer grid cell. The calculation of the dynamic forces $N_{q,g}^{(s)}$ defined in (25) transmitted by all couplings q inside and outside rod segments s on the all level of spacer grids g makes possible to identification of the maximal loaded spacer grid cell.

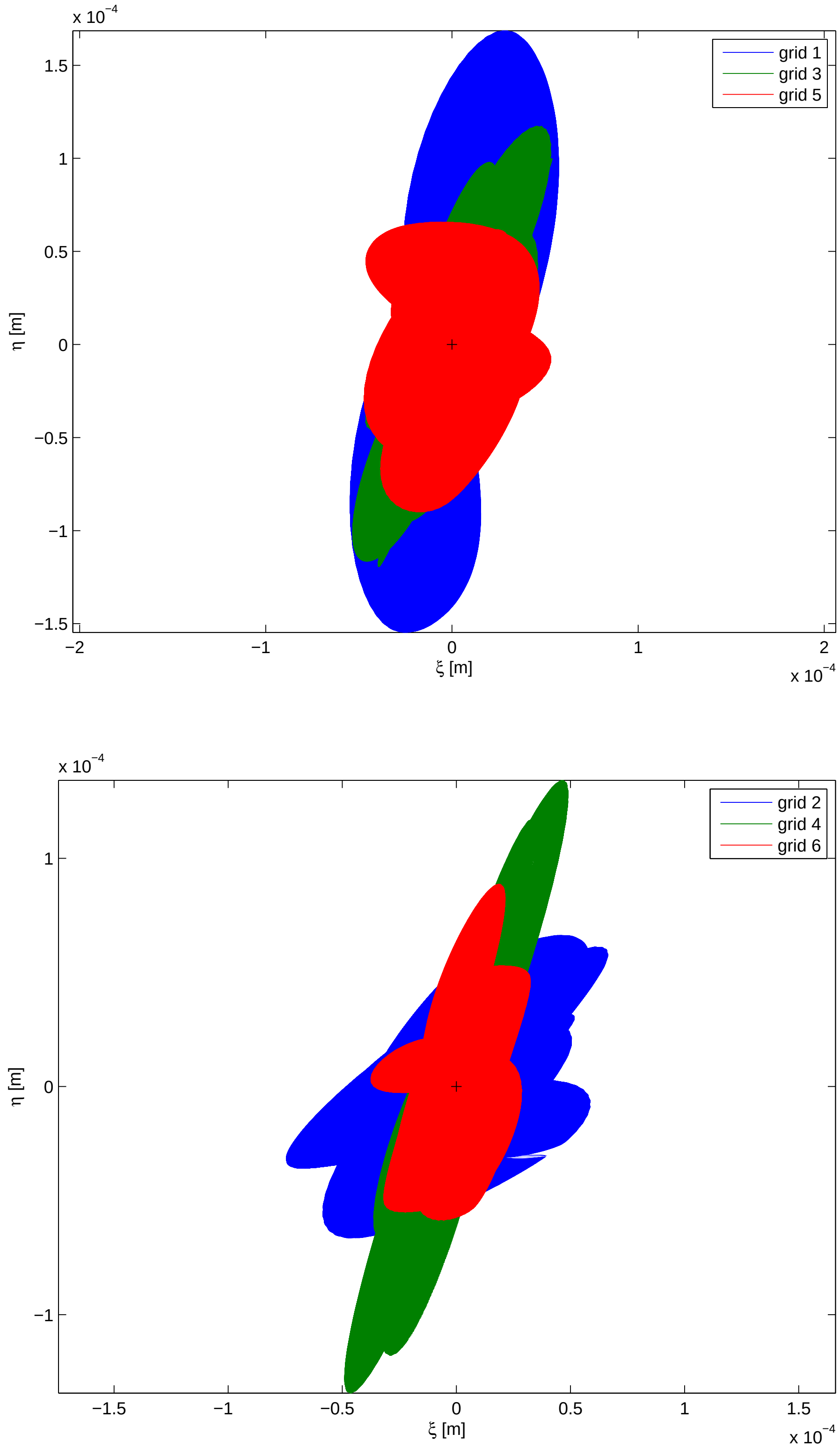

Fig. 5. Orbits of the fuel rod centre $r = 10$ in the first rod segment on the level spacer grids 1, 3, 5 (upper figure) and 2, 4, 6 (lower figure)

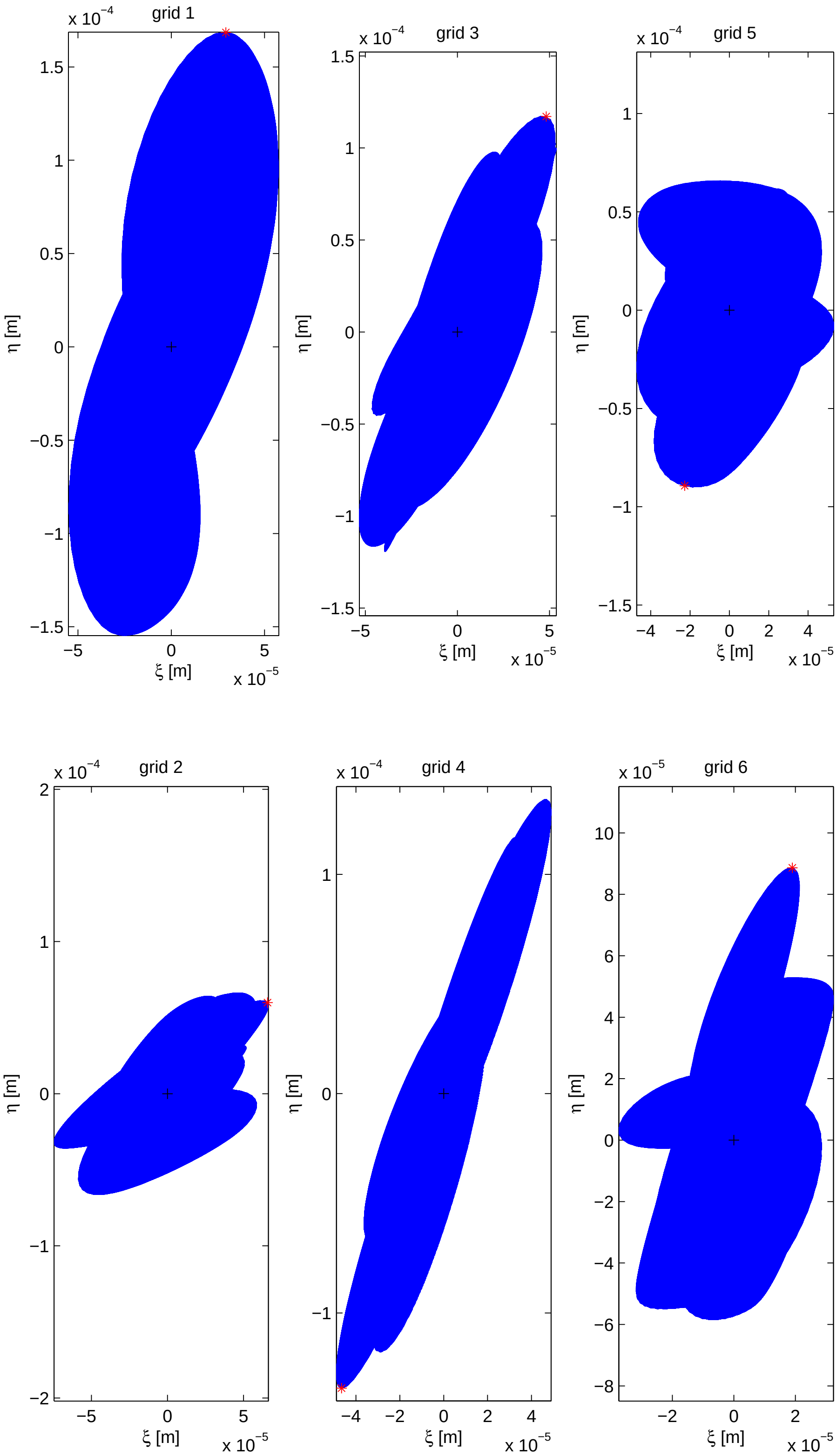

Fig. 6. Orbits from Fig. 5 depicted separately

4. Application

The presented methodology was applied for steady polyharmonic response of the Russian TVSA-T fuel assembly in the VVER 1000 reactor core in NPP Temelín. As an illustration, the orbits in transverse planes of the random selected ($r = 14$) fuel rod centre in the first fuel rod segment ($s = 1$) on the level spacer grids $g = 1$ to 6 caused by pressure pulsations generated by all circulation pumps [11] are shown in Fig. 5 and separately in Fig. 6. The revolution frequencies of the particular pumps in some coolant loops are slightly different $f_1 = f_2 =$ 16,635 Hz and $f_3 = f_4 =$ 16,645 Hz, whereas three harmonic components ($k = 1, 2, 3$) of the pressure pulsations were respected. The condensed model (20) with 3272 DOF ($m_s = 500, m_{CT} = n_{CT} = 32, m_{LS} = n_{LS} = 240$) was used for the calculation of the orbits. The accuracy of condensed model was tested in terms of relative errors of 125 lowest fuel assembly eigenfrequencies defined in the form

$$\varepsilon_\nu = \frac{|f_\nu(m_s) - f_\nu|}{f_\nu}, \quad \nu = 1, \ldots, 125\,, \tag{30}$$

where f_ν are eigenfrequencies of the full (noncondensed) model with 10832 DOF. The relative errors ε_ν for different condensation level of the rod segments expressed by number of the rod segment master eigenvectors $m_s = 100, 300, 500$ were investigated in [14]. Relative errors decrease with decreasing condensation level (m_s increases) in all FA eigenfrequencies. Upper limit of the relative error for $m_s = 500$ is in some higher eigenfrequencies 6 %. The orbits of these particular models distinguish only little. Time behaviour of the dynamic force transmitted by maximal loaded spacer grid cell (coupling $q =$147 between fuel rod 6 in segment 3 and fuel rod 46 in segment 4) on the level of the first spacer grid ($g = 1$) is demonstrated in the Fig. 7. Time behaviour of the slip speed and friction power in the contact points of the mentioned full rods with spacer grid cell is shown in Fig. 8 (slip speed) and Fig. 9 (friction power).

5. Conclusion

The described method in direct sequence at the fuel assembly conservative mathematical model derived in [13] enables to investigate the flexural kinematic excited vibrations of all FA components. The vibrations are caused by spatial motion of the two horizontal support plates in the reactor core transformed into displacements of the kinematical excited nods of the FA components-fuel rods, guide thimbles, centre tube and skeleton angle pieces. The special coordinate system of radial and orthogonal lateral and flexural angular displacements around these directions of the fuel rods and guide thimbles enables to separate the hexagonal type FA into six identical revolved rod segments characterized in global FA mathematical model by identical mass, damping and stiffness matrices. These identical subsystems are linked each other and with centre tube and skeleton by spacer grids on the several level. All FA components are modelled as one dimensional continuum of beam type with nodal points in the gravity centres of their cross-sections on the level of the spacer grids.

The FA mathematical model has, in consequence of great number of fuel rods, to large DOF number for calculation of the dynamic response. Therefore is compiled condensed model based on reduction of the number rod segment eigenvectors conducive to FA dynamic response using modal synthesis method. The developed methodology was used for steady vibration analysis of the Russian type nuclear FA caused by motion of the support plates, excited by pressure pulsations generated by main circulation pumps in the coolant loops of the primary circuit. The

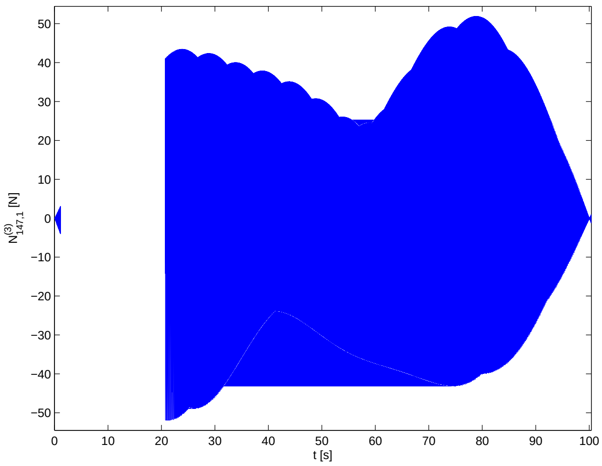

Fig. 7. Dynamic force transmitted by maximal loaded spacer grid cell (coupling 147, spacer grid 1)

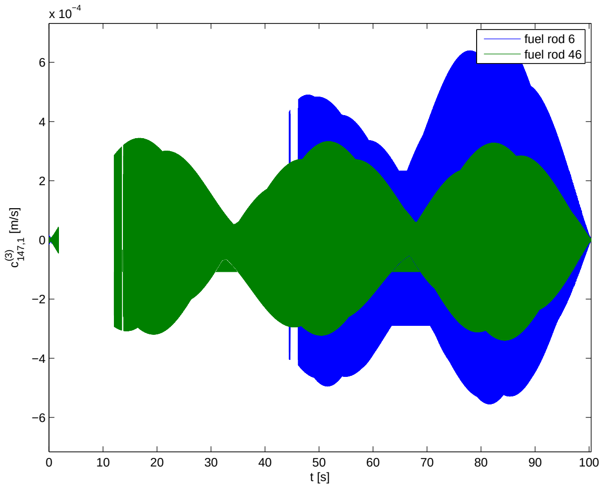

Fig. 8. Slip speed in the contact points specify in Fig. 7

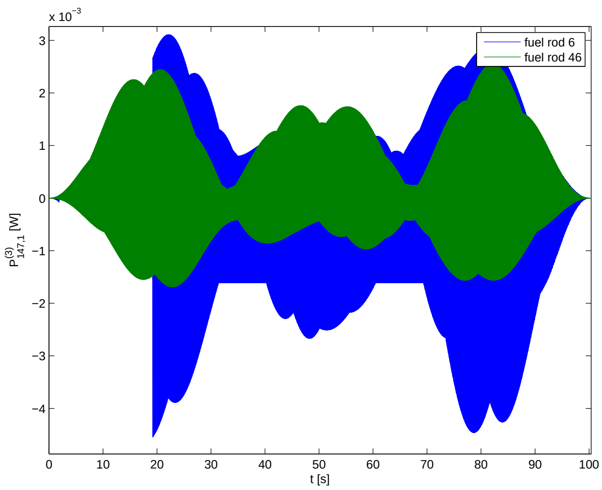

Fig. 9. Friction power in the contact points specify in Fig. 7

developed software in MATLAB is conceived so, that enables to choose an arbitrary configuration of operating pumps whose rotational frequencies slightly differentiate in the experimentally determined frequency interval $f \in \langle 16.635; 16.645\rangle$ Hz. This phenomenon implicates beat vibrations, which amplify dynamic normal and friction forces in the contact of the fuel rod coating and spacer grid cells. The software enables an identification of the maximal dynamic loaded spacer grid cell and calculation of the maximal friction force work during defined time period. In this way the abrasion of fuel rod coating can be estimated.

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