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The mean stress influence on lifetime under high-cycle fatigue combined loading

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Abstract

This paper summarizes the results from the field of combined loading. This study builds on experiments that have been performed during the last ten years and provides information on sinusoidal in-phase stress components and tenseness distribution. Unfortunately, there is no exact method for estimating the damage caused by already known stress history. The main aim of experimental works was to determine the analytical form of Haigh diagrams and analytical expression of terminal lines equations. This article deals with different combinations of tension and torsion pre-stresses and contains results of experiments carried out in the laboratory of Institute of Thermomechanics in Pilsen. The lifetime curves (S-N curves) for combined pre-stress for different damage effect of normal and shear stress components have been obtained. The final results have been compiled and displayed at the three-dimensional Haigh diagram with normalized coordinates. The terminal lines equations are solved by non-linear regression by Fletcher's version of the Levenberg-Maquardt algorithm for the minimization of a sum of squares of equation residuals.

Keywords: fatigue, combined loading, mean stress

1. Introduction

Questions of material lifetime are still not quite solved for combined loading. Multiaxial fatigue has been in the centre of attention all over the world in the last decades [1]. Simultaneously new data-processing algorithms had to be developed. Unfortunately, there is no exact method for estimating the damage caused by already known stress history. Hence, a great number of hypotheses have raised up during the recent 90 years. Generally, the phenomenological fatigue theories can be classified into three categories, stress-based, strain-based and energy-based, depending on the kind of chosen damage parameter which can be a function of stress, strain or strain energy density [2]. All the three approaches are mentioned and compared in paper [2]. The influence of mean values in both stress components of combined loads belongs among not quite solved problems and sufficient information on material behaviour misses for this loading type. It has been found, in the low cycle fatigue regime of ductile metal materials, that an introduction of mean strain usually does not appreciably affects the fatigue life because of the mean stress relaxation caused by plastic deformation during the early life [2]. However, mean stress $s_m = (s_{\max} + s_{\min})/2$ [3] plays an important role in the damaging process during the high cycle fatigue. It changes the value of fatigue limit s_c and corresponding fatigue life (cycle count on fatigue limit) N_c. At zero mean stress, the allowable stress amplitude is the effective

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fatigue limit for a specified fatigue life. As the mean stress increases, the permissible stress amplitude decreases. The allowable cyclic amplitude is zero [3] at a mean stress equal to the ultimate tensile strength limit of the material.

If the mean value is higher, the damage produced by maximal stresses in cycle would be higher too [4], and therefore the positive mean stress decreases the fatigue limit. The lifetime should prolong in the case of negative normal pre-stress because of its closing effect on the crack head. This hypothesis was published in [5] without an experimental confirmation. The influence appears systematically, therefore s_{cm} has been established which denotes s_c as a function of mean stress, $s_{cm} = s_c(s_m)$, and $N_{cm} = N_c(s_m)$. The graphical illustration of harmonic cycles is called Haigh diagram. The aim of this study was to determine Haigh diagrams in analytical form under combined loading by components of normal and shear stress.

2. Current state of problems

The generation of Haigh diagram is very expensive and time-consuming because every point of the terminal line represents one Wöhler curve. That is a reason why only a few points are fitted by an approximate regression line. Nevertheless, the dependence is general, therefore the following analytical function is a good approximation in certain occurrences

$$s_{cm} = s_c \left[1 - \frac{s_m}{s_F}\right]^{k_H}, \tag{1}$$

where s_F is so called fictive stress. It is used to determine experimentally coefficient k_H. For $k_H = 1$, the previous function becomes a straight line that goes through the points $(0, s_c)$ and $(s_F, 0)$, where s_F is often defined as strength limit R_m. If we want to obtain the safe area of tenseness we must accomplish intersection of this line with line $s_m + s_a = R_e$, where the amplitude is done as $s_a = (s_{\max} - s_{min})/2$ [3]. The constructions proposed in this way should not show any fatigue failure.

There are five different possibilities how to apply loading as a combination of tension and torsion. These possibilities are based on the general equation for complex damaging stress

$$\sigma_d(t) = \sigma_m + ik_c\tau_m + \sigma_a \sin(2\pi f_z t) + ik_c\tau_a \sin(2\pi f_z t + \varphi), \tag{2}$$

where the constant k_c is a ratio of fatigue limits in tension-pressure and in shear without mean values, σ_m and τ_m are normal and shear mean values, σ_a and τ_a are normal and shear amplitudes, t is time of loading and f_z is loading frequency and φ is a phase angle. All possibilities are shown in Table 1.

Table 1. Table of nomenclature for combined tension and torsion test

$\sigma_{cm} = \sigma_c(\sigma_m)$	uniaxial tensile fatigue limit with pre-stress
$\tau_{cm} = \tau_c(\tau_m)$	uniaxial torque fatigue limit with pre-stress
$\sigma_{c\tau} = \sigma_c(\tau_m)$	biaxial fatigue limit with dynamical normal component and with static pre-stress in torsion
$\tau_{c\sigma} = \tau_c(\sigma_m)$	biaxial fatigue limit with dynamical torsional component and with static pre-stress in tension/pressure
$\sigma_{cm} = k_c\tau_{cm} = \sigma_c(\sigma_m, \tau_m)$	biaxial fatigue limit with static pre-stress in both stress components and with one dynamical component.

The direction of the crack propagation is controlled only by the character of the stress component [6]. The static component has an influence on crack propagation speed because in the case of positive pre-stress crack front opens and it affects the overall life.

3. Experimental program

The results published in [7] provide information on sinusoidal in-phase stress components and tenseness distribution. In our conception, except for normal component, experiments were extended by attaching a shear stress component in the form of static pre-stress. All experiments were carried out with uniaxial sinusoidal 10 Hz loading.

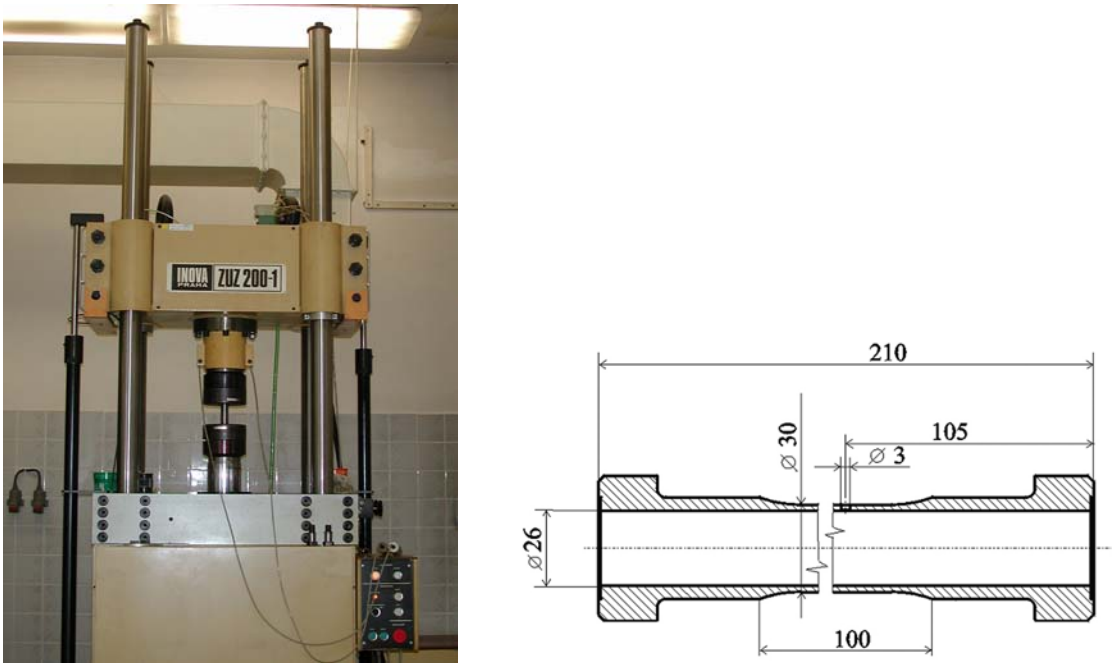

Fig. 1. Servohydraulic fatigue testing machine and dimensions of the specimen

The experiments were carried out on an electrohydraulic computer-controlled testing machine Inova ZUZ 200-1 (see Fig. 1). This type of machine is able to reach the maximal force in tension-pressure up to 200 kN and simultaneously the torque moment 1 kNm. The machine is equipped with mechanical jaws. The specimens were tubular with 30 mm diameter, wall thickness 2 mm and a notch in the shape of one-sided transverse hole of diameter 3 mm positioned in the middle of specimen. The specimens were made of structural low carbon ČSN 41 1523.1 steel after normalization annealing. The American equivalent of this material type is ASTM A623 [8]. Its chemical composition is mentioned in Table 2.

Table 2. Table of chemical composition of the specimen [8]

C	Si	Mn	P	S	Cu
max 0.2	max 0.55	max 1.6	max 0.025	max 0.025	max 0.55

This study builds on experiments that have been performed at workplace of Institute of Thermomechanics in Pilsen during the last ten years. It was necessary to determine the fatigue limit in tension-pressure and in torsion. The step test method similar to [9] was used for the evaluation of the fatigue limit. This method determinates the fatigue limit with a 90 % band of reliability. The data were experimentally established from 9 specimens as fatigue limit value 120 MPa in tension-pressure and from 14 specimens as value 80 MPa in torsion. The ratio of these values gives the coefficient $k_c = 1.5$ that informs on damage effect distribution between normal and shear components. The experiment of combined loading with mean values in normal and shear components was proposed in agreement with the coefficient k_c (the ratio $3 : 2$ of tensile and shear components). The stress mean values $\sigma_m = 150$ MPa and $\tau_m = 100$ MPa were chosen for the first test which provided the level of fatigue limit on 85 MPa from 9 specimens. Cycle count on fatigue limit (breaking point from slant to horizontal branch in Fig. 2) corresponds to 8 550 076. Mean stresses were chosen in agreement with the ratio coefficient k_c. The values resulting from this test are processed in accordance with probabilistic approach in logarithmic axes as Fig. 2 shows. This regression line is called Wöhler curve and the shape of straight line equation of slanted branch for combined pre-stress is defined by an equation that was found by probabilistic approach as

$$\log N_a = 31.845\,4 - 12.912\,4 \log \sigma_a. \tag{3}$$

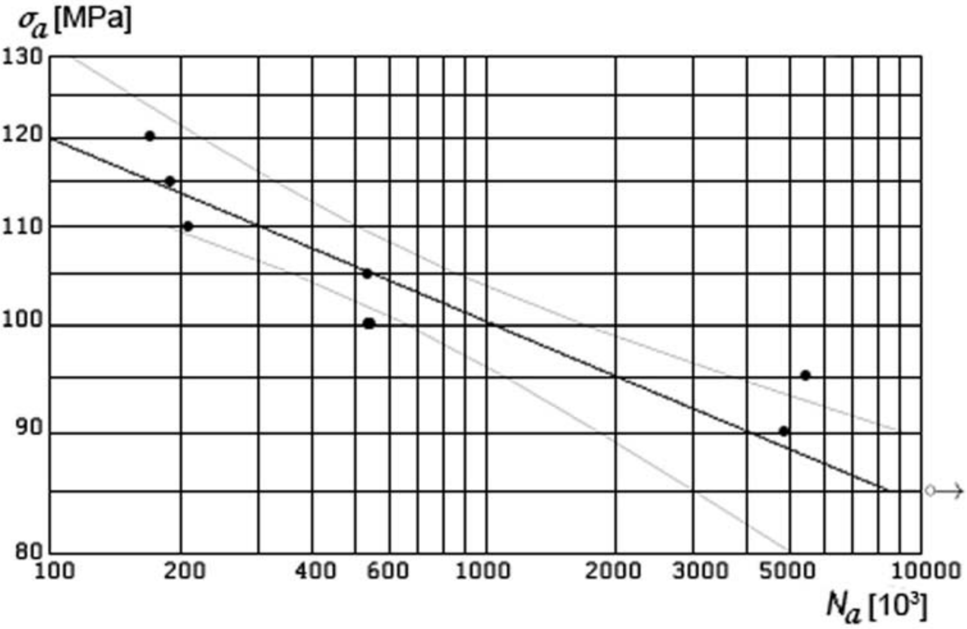

Fig. 2. Wöhler curve for low carbon steel ČSN 41 1523.1 with applied combined pre-stress in normal component $\sigma_m = 150$ MPa and in shear component $\tau_m = 100$ MPa

Another experiment with combined loading kept the same ratio of static pre-stresses with mean values $\sigma_m = 200$ MPa and $\tau_m = 133.3$ MPa and was performed on 9 specimens. The output from all these results was the level of fatigue limit about magnitude 45 MPa. It can be seen in Wöhler curve in Fig. 3 where the curve crosses into horizontal line. Point scatter is relatively small in this case. Cycle count on fatigue limit (breaking point from slant to horizontal branch in Fig. 3) corresponds to 8 887 917. The formula for slanted branch of Wöhler curve was determined by the linear regression in the form:

$$\log N_a = 14.383\,6 - 4.497\,2 \log \sigma_a. \tag{4}$$

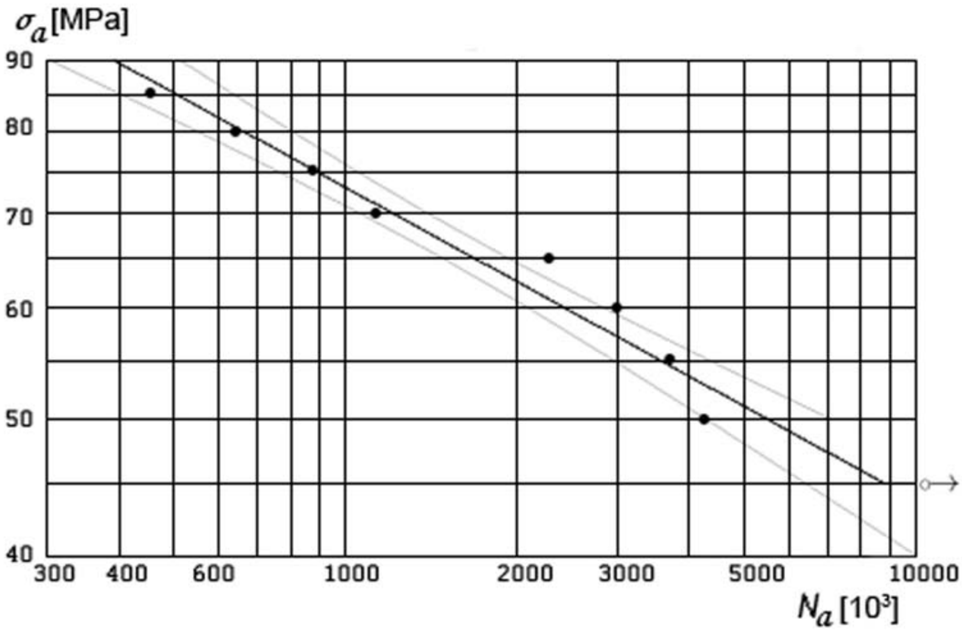

Fig. 3. Wöhler curve for low carbon steel ČSN 41 1523.1 with applied combined pre-stress in normal component $\sigma_m = 200$ MPa and in shear component $\tau_m = 133.3$ MPa

4. Evaluation of results

If the slanted branches of both curves in Fig. 2 and Fig. 3 are compared, it can be found out that the directions are considerably different in spite of the fact that herein [10] the results are mentioned on the basis of hypothesis that the direction is a material constant. One of the possible explanations is that the yield point was exceeded in the second experiment. According to [11] the yield point of this material is about 345 MPa. If the complex damaging stress is computed only for pre-stress components, it can be stated that this value was really exceeded.

Results from the previous experiments published in [6] were used for the complementation of the three-dimensional Haigh diagram. An experiment with combined pre-stresses $\sigma_m = 150$ MPa and $\tau_m = 100$ MPa has resulted in the fatigue limit $\sigma_{cm} = 85$ MPa, and another one with values $\sigma_m = 200$ MPa and $\tau_m = 133.3$ MPa has resulted in the fatigue limit $\sigma_{cm} = 45$ MPa. The results from these experiments were added to the previous data from [6]. The next point of Haigh diagram is the strength limit in tension $R_{m\sigma}$. This value was determined on 550.87 MPa from two specimens. The strength limit in torsion $R_{m\tau}$ can be determined from strength limit in tension by division of constant k_c, when the value 367.25 MPa is obtained. All points are displayed in normalized coordinates in the three-dimensional graph. The curves have been plotted with coefficients obtained by non-linear regressions.

A fitting curve experimental data was obtained by a Fletcher's version of the Levenberg-Maquardt algorithm for the minimization of a sum of squares of equation residuals. The program was created in numerical computing environment and programming language MATLAB. Since the normalized coordinates/data were used, it was necessary to find only one parameter k_H of regression curve. The approximate equation (1) was chosen for this solution. The vector of residuals was defined as an anonymous function in the program. The guess p_0 of a number 0.5 was selected as starting solution. The final result was obtained in a few iterations as a parameter p multiplied by starting guess p_0.

Table 3. The output of iteration process for the minimization of a sum of squares of equation residuals for computation of terminal curve coefficient k_H in the Haigh diagram for points of tensile pre-stresses

Iteration	$\sum r_i^2$	p	dp
1	0.074 929	1	0
2	0.003 065 1	1.766 2	−0.766 2
3	0.002 442 6	1.851 9	−0.085 736
4	0.002 442 5	1.853 2	−0.001 281 1
5	0.002 442 5	1.853 2	−0.000 005 792 6

Table 4. The output of iteration process for the minimization of a sum of squares of equation residuals for computation of terminal curve coefficient k_H in the Haigh diagram for points of shear pre-stresses

Iteration	$\sum r_i^2$	p	dp
1	0.0112 52	1	0
2	0.000 233 28	0.501 8	0.498 2
3	0.000 195 34	0.528 03	−0.026 23
4	0.000 195 34	0.528 03	0.000 006 806 4

Table 5. The output of iteration process for the minimization of a sum of squares of equation residuals for computation of terminal curve coefficient k_H in the Haigh diagram for points of combined pre-stresses in ratio of $3\sigma_m$ to $2\tau_m$

Iteration	$\sum r_i^2$	p	dp
1	0.109 8	1	0
2	0.021 723	1.96	−0.960 04
3	0.020 617	2.105 2	−0.145 15
4	0.020 616	2.103 2	0.002 027 6
5	0.020 616	2.103 2	−0.000 076 153

In the first case the data from previous experiments were summarized, individual points serving as a program input. The output for tensile pre-stresses in the form of iteration number, sums of squares of residuals r_i, computed parameters p and their increments are indicated in Table 3–5. The final value for coefficient k_H of curve in the Haigh diagram for points of tensile pre-stresses was $0.926\,6$. Altogether five iterations were necessary to get the resulting p.

The same process of computation was applied for other two cases of shear pre-stresses and combined components of pre-stress. The coefficient k_H is equal to 0.264 for the terminal curve of shear pre-stress. Particular iterations of this process are presented in Table 4.

The resulting coefficient for combined pre-stress components is $1.051\,6$. Altogether five iterations ran across the program and particular iterations of this process are in Table 5. Fig. 4 shows a graph with all terminal regression curves.

It should be noted that the same computation was performed only for the first gained point of combined pre-stress $\sigma_m = 150$ MPa and $\tau_m = 100$ MPa. If the second point of combined pre-stress was neglected, the terminal curve coefficient k_H equal to $0.709\,1$ would be obtained. This terminal curve would have a more rounded shape. This would have been more understandable

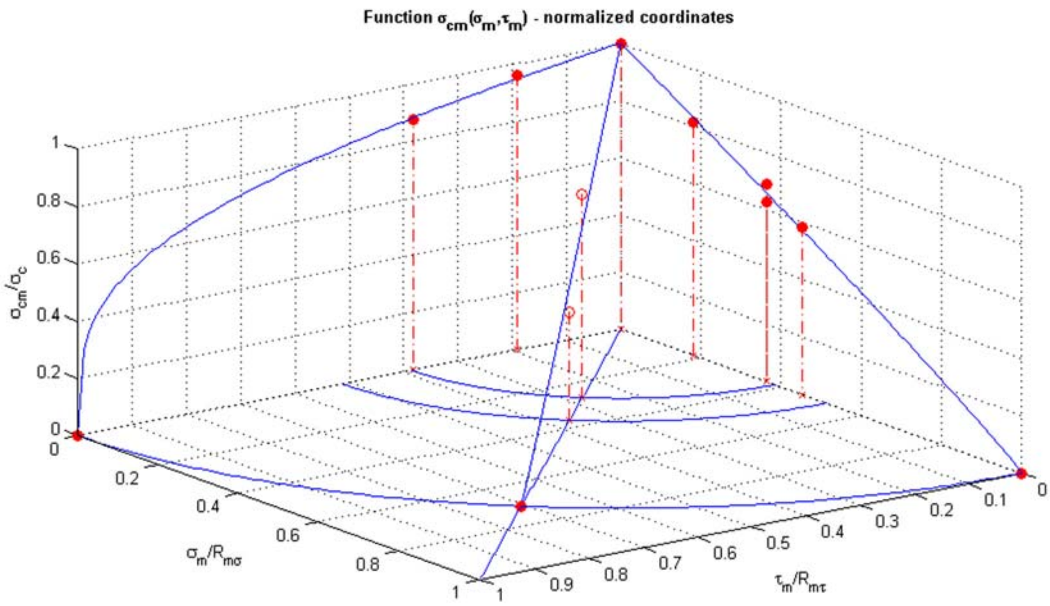

Fig. 4. Haigh diagrams for shear and tensile mean stresses and for their combination in ratio of $3\sigma_m$ to $2\tau_m$ that are plotted in normalized coordinates in the three-dimensional graph

considering all area of tenseness. It could be said about the area of 3D Haigh diagram in Fig. 4 that it crosses from a nearly straight terminal line (Goodman line) for normal pre-stresses where coefficient k_H is getting closer to value one to terminal curve for shear pre-stresses where it is possible to observe a concave curvature. It is possible to declare the following hypothesis for chosen ratio ($k_c = 1.5$) between normal and shear components in case of combined pre-stress. If the shear component of pre-stress predominates, the exponent of terminal curves acquires the values from 0.264 to 0.709. On the contrary, if the normal component of pre-stress predominates, the exponent of terminal curves will acquire the values from $0.709\,2$ to $0.926\,6$.

With regard to the fact that curves are obtained by regression, it is necessary to perceive all the gained values as approximate. It is evident from the graph in Fig. 4 that the second point of combined pre-stresses, $\sigma_m = 200$ MPa and $\tau_m = 133.3$ MPa, lies lower than others points except for strength points and it is the only one for which the yield point of material was exceeded.

5. Conclusion

The aim of this paper was to summarize the results from the field of uniaxial pre-stress and the results with combined pre-stress. The specimens were made from structural low carbon ČSN 41 1523.1 steel after normalization annealing. All points were displayed in the three-dimensional graph and interlaid by terminal lines. For an analytical expression of terminal lines, the non-linear regression was used for equation (1). The program was created in numerical computing environment and programming language MATLAB that uses Fletcher's version of the Levenberg-Maquardt algorithm for the minimization of a sum of squares of equation residuals. Computed exponent k_H for terminal line in the case of static tensile pre-stress was established as $0.926\,6$. In the case of static shear pre-stress k_H was $0.264\,0$. These values were determined on the basis of former results as a constant of ratio of fatigue limit in tension-pressure and in shear without mean value $k_c = 1.5$. Accordingly the ratio $3 : 2$ of tensile and shear pre-stress was chosen. The same procedure was applied on this combined pre-stress, the result for exponent k_H being $1.051\,6$. The selected ratio assures the position in the middle of three-dimensional graph with normalized coordinates.

It can be noticed that the exponent for the case of single tensile pre-stress is going to number one and the terminal curve is almost linear in the Haigh diagrams given in Fig. 4. By contrast in the case of single shear pre-stress the terminal line has a concave curvature with the exponent 0.264. In the case of combined pre-stress, a sequential crossing was presupposed. However the value of fatigue limit with higher mean stresses is very low. In this case the fatigue strength could be reduced when the maximum stress (sum of alternating and mean stresses) exceeds the material yield stress as described in [12]. However, a more accurate description would require more experiments.

These conclusions are only approximate and based on experiments on a particular type of material. Therefore it could be interesting to compare these results with similar results for another material. Experiments of mean stress influence on lifetime could also be interesting for environmental fatigue with the influence of temperature.

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