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Thermal analysis of isotropic plates using hyperbolic shear deformation theory

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Abstract

In this paper, thermal analysis of a thick isotropic rectangular plate is carried out using the hyperbolic shear deformation theory (HYSDT). The displacement field of the theory contains three variables. The hyperbolic sine and cosine functions are used in the displacement field in-terms of thickness coordinate to represent the effect of shear deformation. The most important feature of the theory is that, the transverse shear stresses can be obtained directly from the use of constitutive relations, hence the theory does not need shear correction factor. The theory accounts for parabolic distribution of transverse shear stresses across the thickness satisfying the stress free boundary conditions at top and bottom surfaces of the plate. Governing differential equations and boundary conditions of the theory are obtained using the principle of virtual work. The results obtained for bending analysis of isotropic plates subjected to uniformly distributed thermal load are compared with those obtained by other theories, to validate the accuracy of the presented theory.

Keywords: isotropic plate, shear correction factor, hyperbolic function, transverse shear stress, thermal load

1. Introduction

Isotropic plates are being widely used in structures subjected to severe thermal environment, which produce an intense thermal stresses on it. In order to describe the correct thermal response of thick plates including shear deformation effects, refined theories are required.

Classical plate theory (CPT) is inaccurate for thick plates due to neglect of the transverse shear stress. To overcome limitations of CPT the displacement based first order shear deformation theory (FSDT) is developed by Mindlin [10]. Since transverse shear stress distribution is constant through the thickness in FSDT, it requires shear correction factor. CPT and FSDT are inadequate to predict the accurate solutions of a thick isotropic plate hence the higher order shear deformation theories (HSDT) are developed. Moreover the lot of research is available on thermal analysis of isotropic plate. Reddy [12] has developed simple higher order shear deformation theory for analysis of plates. Sayyad and Ghugal [13, 14] have developed an exponential shear deformation theory (ESDT) for buckling, bending and free vibration analysis of thick isotropic plates. Ghugal and Kulkarni [3] have used trigonometric shear deformation theory (TSDT) for thermal analysis of composite plates.

Kapuria and Achary [6] develop a new zig-zag HSDT to study the thermal response of plates whereas Zhen and Wanji [21] present a global-local higher-order theory for triangular element under a thermal load. Cho and Oh [8] develop a higher order zig-zag plate theory for

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thermo-mechanical loading. Matsunaga [8] has presented a two-dimensional (2D) higher order deformation theory for the evaluation of displacements and stresses in functionally graded (FG) plates subjected to thermal and mechanical loadings. Zenkour [20] presented the static thermo-elastic response of symmetric and anti-symmetric cross-ply laminated plates using the unified shear deformation theory. Wang et al. [19] studies the response histories and the distribution of dynamic interlaminar stresses in laminated plates with complex boundaries, subjected to free vibrations and thermal load, based on dynamic differential equations. Matsunaga [7] has compared 2-D single-layer and 3-D layerwise theories for computing interlaminar stresses of laminated composite. Nguyen and Caron [11] studied the finite element analysis of free-edge stresses of composite laminates subjected to mechanical and thermal loads. Ali et al. [1] develops a new theory for accurate thermal/mechanical flexural analysis of symmetric laminated plates.

There exist another class of refined shear deformation theories in which hyperbolic sine and cosine functions are used in-terms of thickness coordinate to include the effect of transverse shear deformation. Soldatos [16, 17] has used hyperbolic function first time to include the effect of shear deformation for the static and dynamic analysis of the plate under mechanical load. Ghugal and Pawar [4] extended this theory for the buckling and free vibration analysis of orthotropic plates. Metin [9] also presented bending, buckling and free vibration analysis of laminated composite plates using hyperbolic shear deformation theory. Sayyad and Ghugal [15] studied the effect of hyperbolic shear deformation theory on bending analysis of isotropic beams under mechanical loading.

It is pointed out from the above literature that efficiency of hyperbolic shear deformation theory for beams and plates has been proved for mechanical load but bending response of plate under thermal load is not studied by the researchers. Therefore, in the present study, attempt is made to check the efficiency of hyperbolic shear deformation theory for the flexural analysis of isotropic plates under the thermal load linear through the thickness.

2. Mathematical modelling

Mathematical modelling of the plate is based on the following kinematical assumptions:

1. The displacements are small and therefore strains involved are infinitesimal.
2. The inplane displacement u in x direction as well as displacement v in y direction consists of three parts:
   (a) Displacement component analogous to the displacement in classical plate theory of bending.
   (b) Displacement component due to shear deformation, which is assumed to be hyperbolic in nature with respect to thickness coordinate, such that the maximum shear stress occurs at neutral axis.
   (c) The displacements are small compared to plate thickness.
3. The transverse displacement w in z direction is assumed to be a function of x and y coordinates only.
4. The body forces are ignored in the analysis.
5. The plate is subjected to thermal load only.

Using the above assumptions, the theoretical formulation of presented theory for an isotropic plate can be done. Let us consider a rectangular plate in cartesian coordinate system occuping the region given by

$$0 \le x \le a, \quad 0 \le y \le b, \quad -h/2 \le z \le h/2, \tag{1}$$

where x, y, z are the cartesian coordinates, a and b are the edge lengths in the x and y directions respectively, and h is the thickness of the plate. The plate is made up of isotropic material which obeys generalised Hooke's law.

2.1. Displacement field

Based on the assumptions made in the previous section, hyperbolic shear deformation theory (HYSDT) proposed by Soldatos [16, 17] is used for the mathematical formulation. The displacement field of presented theory is as follows:

$$\begin{aligned} u &= -z\,\frac{\partial w}{\partial x} + \left[z\cosh\left(\frac{1}{2}\right) - h\sinh\left(\frac{z}{h}\right)\right]\phi(x,y), \\ v &= -z\,\frac{\partial w}{\partial y} + \left[z\cosh\left(\frac{1}{2}\right) - h\sinh\left(\frac{z}{h}\right)\right]\psi(x,y), \\ w &= w(x,y), \end{aligned} \tag{2}$$

where u and v are the inplane displacement components in the x and y directions respectively, and w is the transverse displacement in the z direction. The hyperbolic function in terms of thickness coordinate in both the displacements u and v is associated with the transverse shear stress distribution through the thickness of the plate and the functions $\phi(x, y)$ and $\psi(x, y)$ are the unknown functions associated with the shear slopes.

2.2. Strain-displacement relationships

Normal and shear strains are obtained within the framework of linear theory of elasticity [18] using the displacement field given by Eq. (2) is as given in Eq. (3)

$$\begin{gathered} \varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \varepsilon_z = \frac{\partial w}{\partial z}, \\ \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad \gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}. \end{gathered} \tag{3}$$

2.3. Stress-strain relationships

The stress-strain relationships considering thermal effect given by Jones [5] can be expressed as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \frac{E}{1-\mu^2} \begin{bmatrix} 1 & \mu & 0 & 0 & 0 \\ \mu & 1 & 0 & 0 & 0 \\ 0 & 0 & (1+\mu)/2 & 0 & 0 \\ 0 & 0 & 0 & (1+\mu)/2 & 0 \\ 0 & 0 & 0 & 0 & (1+\mu)/2 \end{bmatrix} \begin{Bmatrix} \varepsilon_x - \alpha_x T \\ \varepsilon_y - \alpha_y T \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}, \tag{4}$$

where $(\sigma_x, \sigma_y, \tau_{xy}, \tau_{yz}, \tau_{zx})$ are the stress components, $(\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{yz}, \gamma_{zx})$ are the strain components, E is the modulus of elasticity, μ is the Poisson's ratio and α_x and α_y are the coefficients of thermal expansion along x and y direction respectively. T is the thermal load. Assuming linear variation of temprature through the thickness of the plate [3], the value of thermal load is given as $T = zT_1(x, y)$, where, z is the thickness coordinate and T_1 is the transverse temperature load.

3. Governing equations and boundary conditions

The governing equation of euilibrium can be derived using the principal of virtual displacements. The analytical form of principle of virtual work is as follows:

$$\int_{z=-h/2}^{z=h/2}\int_{y=0}^{y=b}\int_{x=0}^{x=a}[\sigma_x\delta\varepsilon_x+\sigma_y\delta\varepsilon_y+\sigma_z\delta\varepsilon_z+\tau_{yz}\delta\gamma_{yz}+\tau_{zx}\delta\gamma_{zx}+\tau_{xy}\delta\gamma_{xy}]\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z-$$
$$\int_{x=0}^{x=a}\int_{y=0}^{y=b} q(x,y)\,\delta w\,\mathrm{d}x\,\mathrm{d}y=0, \tag{5}$$

where the symbol δ denotes the variation operator and $q(x, y)$ represents a transverse mechanical load. Substituting stresses and strains from Eqs. (3) and (4) into the Eq. (5) and employing Green's theorem successively, we obtain the coupled Euler-Lagrange equations, which are the governing differential equations of the plate and the associated boundary conditions of the plate. The governing differential equations in-terms of stress resultants are as follows:

$$\begin{aligned}\frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_y}{\partial y^2}+q&=0,\\ \frac{\partial N_{sx}}{\partial x}+\frac{\partial N_{sxy}}{\partial y}-N_{Tcx}&=0,\\ \frac{\partial N_{sy}}{\partial y}+\frac{\partial N_{sxy}}{\partial x}-N_{Tcy}&=0.\end{aligned} \tag{6}$$

The boundary conditions at $x = 0$ and $x = a$ obtained are of the following form:

$$\left.\begin{array}{ll}\text{Either } V_x=0 & \text{or } w \text{ is prescribed}\\ \text{Either } M_x=0 & \text{or } \frac{\partial w}{\partial x} \text{ is prescribed}\\ \text{Either } N_{sx}=0 & \text{or } \phi \text{ is prescribed}\\ \text{Either } N_{sxy}=0 & \text{or } \psi \text{ is prescribed}\end{array}\right\}. \tag{7}$$

The boundary conditions at $y = 0$ and $y = b$ obtained are of the following form:

$$\left.\begin{array}{ll}\text{Either } V_y=0 & \text{or } w \text{ is prescribed}\\ \text{Either } M_y=0 & \text{or } \frac{\partial w}{\partial y} \text{ is prescribed}\\ \text{Either } N_{sxy}=0 & \text{or } \phi \text{ is prescribed}\\ \text{Either } N_{sy}=0 & \text{or } \psi \text{ is prescribed}\end{array}\right\}. \tag{8}$$

The reaction at the corners of the plate is of the following form:

$$\text{Either } M_{xy}=0 \quad \text{or } w \text{ is prescribed,} \tag{9}$$

where (M_x, M_y, M_{xy}) are the moment resultants associated with the classical plate theory, $(N_{sx}, N_{sy}, N_{sxy})$ are the refined moment resultants associated with the shear deformation and (N_{Tcx}, N_{Tcy}) are the shear force resultants given as below:

$$\left.\begin{aligned}(M_x, M_y, M_{xy}) &= \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}) z \,\mathrm{d}z; \; (N_{sx}, N_{sy}, N_{sxy}) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}) f(z) \,\mathrm{d}z; \\ (N_{Tcx}, N_{tcy}) &= \int_{-h/2}^{h/2} (\tau_{zx}, \tau_{yz}) \frac{\mathrm{d}f(z)}{\mathrm{d}z} \,\mathrm{d}z; \; V_x = \frac{\partial M_x}{\partial x} + 2\frac{\partial M_{xy}}{\partial y}; \; V_7 = \frac{\partial M_y}{\partial y} + 2\frac{\partial M_{xy}}{\partial x}\end{aligned}\right\}. \quad (10)$$

Substituting Eq. (10) into the Eq. (6), the governing differential equations obtained in-terms of unknown displacement variables used in the displacement field (w, ϕ and ψ) are as follows:

$$\begin{aligned}\delta w\colon \quad & \left(D_{11}\frac{\partial^4 w}{\partial x^4} + (2D_{12} + 4D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4}\right) - \\ & \left(S_{11}\frac{\partial^3 \phi}{\partial x^3} + S_{22}\frac{\partial^3 \psi}{\partial y^3}\right) - (S_{12} + 2S_{66})\left(\frac{\partial^3 \phi}{\partial x \partial y^2} + \frac{\partial^3 \psi}{\partial x^2 \partial y}\right) + \\ & (TD_{11} + TTD_{12})\frac{\partial^2 T_1}{\partial x^2} + (TD_{12} + TTD_{22})\frac{\partial^2 T_1}{\partial y^2} = q, \\ \delta\phi\colon \quad & S_{11}\frac{\partial^3 w}{\partial x^3} + (S_{12} + 2S_{66})\frac{\partial^3 w}{\partial x \partial y^2} - \left(SS_{11}\frac{\partial^2 \phi}{\partial x^2} + SS_{66}\frac{\partial^2 \phi}{\partial y^2}\right) + \\ & C_{55}\phi - (SS_{12} + SS_{66})\frac{\partial^2 \psi}{\partial x \partial y} + (TS_{11} + TTS_{12})\frac{\partial T_1}{\partial x} = 0, \quad (11) \\ \delta\psi\colon \quad & S_{22}\frac{\partial^3 w}{\partial y^3} + (S_{12} + 2S_{66})\frac{\partial^3 w}{\partial x^2 \partial y} - \left(SS_{66}\frac{\partial^2 \psi}{\partial x^2} + SS_{12}\frac{\partial^2 \psi}{\partial y^2}\right) + C_{44}\psi - \\ & (SS_{12} + SS_{66})\frac{\partial^2 \phi}{\partial x \partial y} + (TS_{12} + TTS_{22})\frac{\partial T_1}{\partial y} = 0.\end{aligned}$$

The associated consistent boundary conditions obtained are as below:
Along the edge $x = 0$ and $x = a$

$$\begin{aligned}& -D_{22}\frac{\partial^3 w}{\partial y^3} - (D_{12} + 4D_{66})\frac{\partial^3 w}{\partial y \partial x^2} + \\ & \left(2S_{66}\frac{\partial^2 \psi}{\partial x^2} + S_{22}\frac{\partial^2 \psi}{\partial y^2}\right) + (S_{12} + 2S_{66})\frac{\partial^2 \phi}{\partial x \partial y} - \\ & (TD_{12} + TTD_{22})\frac{\partial T_1}{\partial y} = 0 \quad \text{or } w \text{ is prescribed}, \\ & \left(D_{12}\frac{\partial^2 w}{\partial x^2} + D_{22}\frac{\partial^2 w}{\partial y^2}\right) - S_{12}\frac{\mathrm{d}\phi}{\mathrm{d}x} - S_{22}\frac{\mathrm{d}\psi}{\mathrm{d}y} + \\ & (TD_{12} + TTD_{22})T_1 = 0 \quad \text{or } \frac{\partial w}{\partial x} \text{ is prescribed}, \\ & SS_{66}\left(\frac{\mathrm{d}\phi}{\partial y} + \frac{\mathrm{d}\psi}{\partial x}\right) - 2S_{66}\frac{\partial^2 w}{\partial y \partial x} = 0 \quad \text{or } \phi \text{ is prescribed}, \\ & -S_{12}\frac{\partial^2 w}{\partial x^2} + SS_{12}\frac{\partial \phi}{\partial x} - S_{22}\frac{\partial^2 w}{\partial y^2} + SS_{22}\frac{\partial \psi}{\partial y} - \\ & (TS_{12} + TTS_{22})T_1 = 0 \quad \text{or } \psi \text{ is prescribed}. \quad (12)\end{aligned}$$

Along the edge $y = 0$ and $y = b$:

$$
\begin{aligned}
-D_{11}\frac{\partial^3 w}{\partial x^3} - (D_{12} + 4D_{66})\frac{\partial^3 w}{\partial x \partial y^2} + \\
\left(2S_{66}\frac{\partial^2 \phi}{\partial y^2} + S_{11}\frac{\partial^2 \phi}{\partial x^2}\right) + (S_{12} + 2S_{66})\frac{\partial^2 \psi}{\partial x \partial y} - \\
(TD_{11} + TTD_{12})\frac{\partial T_1}{\partial x} &= 0 \quad \text{or } w \text{ is prescribed,} \\
\left(D_{11}\frac{\partial^2 w}{\partial x^2} + D_{12}\frac{\partial^2 w}{\partial y^2}\right) - S_{11}\frac{\mathrm{d}\phi}{\mathrm{d}x} - \\
S_{12}\frac{\mathrm{d}\psi}{\mathrm{d}y} + (TD_{12} + TTD_{11})T_1 &= 0 \quad \text{or } \frac{\partial w}{\partial y} \text{ is prescribed,} \\
-\left(S_{11}\frac{\partial^2 w}{\partial x^2} + S_{12}\frac{\partial^2 w}{\partial y^2}\right) + SS_{11}\frac{\mathrm{d}\phi}{\mathrm{d}x} + \\
SS_{12}\frac{\mathrm{d}\psi}{\mathrm{d}y} - (TS_{11} + TTS_{12})T_1 &= 0 \quad \text{or } \phi \text{ is prescribed,} \\
SS_{66}\left(\frac{\mathrm{d}\psi}{\mathrm{d}x} + \frac{\mathrm{d}\phi}{\mathrm{d}y}\right) - 2S_{66}\frac{\partial^2 w}{\partial x \partial y} &= 0 \quad \text{or } \psi \text{ is prescribed.}
\end{aligned} \tag{13}
$$

Thus, the variationally consistent governing differential equations and boundary conditions are obtained. The coefficients appearing in the governing differential equations and boundary conditions are given in Appendix. The flexural behaviour of the plate is described by the solution satisfying these equations and the associated boundary conditions at each edge of the plate.

4. Illustrative examples

To assess the performance of the present theory in the prediction of bending response of a plate under a thermal load, a simply supported isotropic rectangular plate of length a, width b, and thickness h is considered. The plate is subjected to uniformly distributed thermal load given by Eq. (14):

$$
T_1 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} T_{1mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right), \tag{14}
$$

where T_{1mn} is the coefficients of Fourier expansion

$$
\begin{aligned}
T_{1mn} &= \tfrac{16T_0}{mn\pi^2} && \text{for } m = 1, 3, 5, \ldots \text{ and } n = 1, 3, 5, \ldots, \\
T_{1mn} &= 0 && \text{for } m = 2, 4, 6, \ldots \text{ and } n = 2, 4, 6, \ldots
\end{aligned} \tag{15}
$$

Here, T_0 is the intensity of thermal load. Following material properties of a steel plate are used to obtained numerical results:

$$
\mu = 0.3, \qquad E = 210 \text{ GPa}, \qquad \alpha_x = \alpha_y. \tag{16}
$$

4.1. The closed – form solution (Navier Solution)

The following solution form for $w(x,y)$, $\phi(x,y)$ and $\psi(x,y)$ is assumed which satisfy the boundary conditions of simply supported plate.

$$\begin{aligned} w(x,y) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{mn} \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right), \\ \phi(x,y) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \phi_{mn} \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right), \\ \psi(x,y) &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \psi_{mn} \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right), \end{aligned} \tag{17}$$

where w_{mn}, ϕ_{mn} and ψ_{mn} are the unknown coefficients, which can be easily valuated after the substitution of Eq. (17) and Eq. (14) in the set of three governing differential equations Eq. (11) and solving the resulting simultaneous equations as,

$$\begin{aligned} K_{11}w_{mn} + K_{12}\phi_{mn} + K_{13}\psi_{mn} &= f_1, \\ K_{21}w_{mn} + K_{22}\phi_{mn} + K_{23}\psi_{mn} &= f_2, \\ K_{31}w_{mn} + K_{32}\phi_{mn} + K_{33}\psi_{mn} &= f_3, \end{aligned} \tag{18}$$

where

$$\begin{aligned} K_{11} &= D_{11}\frac{m^4\pi^4}{a^4} + 2D_{12}\frac{m^2n^2\pi^4}{a^2b^2} + D_{22}\frac{n^4\pi^4}{b^4} + 4D_{66}\frac{mn^2\pi^3}{ab^2}, \\ K_{12} &= -S_{12}\frac{m^3\pi^3}{a^3} - (S_{12}+2S_{66})\frac{mn^2\pi^3}{ab^2}, \\ K_{13} &= -S_{22}\frac{n^3\pi^3}{b^3} - (S_{12}+2S_{66})\frac{m^2n\pi^3}{a^2b}, \\ K_{22} &= SS_{11}\frac{m^2\pi^2}{a^2} + SS_{66}\frac{n^2\pi^2}{b^2} + C_{55}, \\ K_{23} &= (SS_{12}+SS_{66})\frac{mn\pi^2}{ab}, \\ K_{33} &= SS_{22}\frac{n^2\pi^2}{b^2} + SS_{66}\frac{m^2\pi^2}{a^2} + C_{44}, \\ F_1 &= (D_{12}\alpha_x + D_{22}\alpha_y)\frac{n^2\pi^2}{b^2} + (D_{11}\alpha_x + D_{12}\alpha_y)\frac{m^2\pi^2}{a^2}, \\ F_2 &= -(S_{11}\alpha_x + S_{12}\alpha_y)\frac{m\pi}{a}, \\ F_3 &= (S_{12}\alpha_x + S_{22}\alpha_y)\frac{n\pi}{b}. \end{aligned} \tag{19}$$

5. Numerical results

In this paper, the displacements and thermal stresses are determined for simply supported isotropic rectangular plates subjected to uniformly distributed thermal load presented in following non-dimentional form:

$$(\bar{u}, \bar{v}) = \frac{(u, v)}{\alpha_x T_0 b^2}, \qquad \bar{w} = \frac{w}{\alpha_x T_0 b^2 h},$$
$$(\bar{\sigma}_x, \bar{\sigma}_y, \bar{\tau}_{xy}) = \frac{(\sigma_x, \sigma_y, \tau_{xy})}{\alpha_x T_0 E b^2}, \qquad (\bar{\tau}_{yz}, \bar{\tau}_{xz}) = \frac{(\tau_{yz}, \tau_{xz})}{\alpha_x T_0 E b}. \tag{20}$$

6. Discussion of results

To validate the efficiency of the presented theory HYSDT, numerical results are compared with those obtained by TSDT of Ghugal and Kulkarni [3], FSDT of Mindlin [10] and CPT. Numerical results of non-dimensional displacements and stresses are presented in Tables 1–3 and Figs. 1–5. Examination of Table 1 reveals that the non-dimensional displacements and stresses obtained by the presented theory are identical with those obtained by TSDT [3] for aspect ratios 5 and 10. It is also pointed out from the numerical results shown in Table 1 that the maximum value of transverse shear stresses are zero for isotropic plates subjected to thermal load. The through thickness variation of in-plane displacement shown in Fig. 1 is observed to be linear. The through thickness variation of in-plane stresses are shown in Figs. 2–3. Table 2 and Fig. 4 show that, the non-dimensional transverse displacement increases with respect to increasing aspect ratio (b/h).

Table 1. Comparison of displacements and stresses for the square isotropic plate subjected to uniformly distributed thermal load

b/h	Source	Model	$\bar{u}$	$\bar{v}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{zx}$	$\bar{\tau}_{yz}$
5	Present	HYSDT	0.0429	0.0429	0.4789	0.0525	0.0525	0.2406	0.00	0.00
	Ref. [3]	TSDT	0.0429	0.0429	0.4789	0.0525	0.0525	0.2406	0.00	0.00
10	Present	HYSDT	0.0215	0.0215	0.9579	0.0262	0.0262	0.1203	0.00	0.00
	Ref. [3]	TSDT	0.0215	0.0215	0.9579	0.0262	0.0262	0.1203	0.00	0.00
	Kirchoff	CPT	0.0215	0.0215	0.9579	0.0262	0.0262	0.1203	0.00	0.00

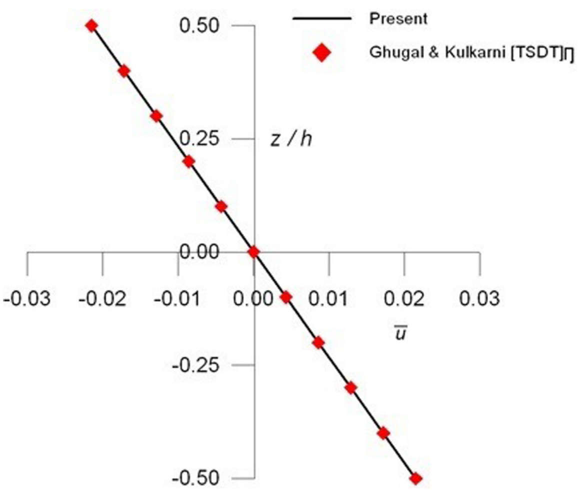

Fig. 1. Through thickness variation of in-plane displacement $\bar{u}$ for aspect ratio 10

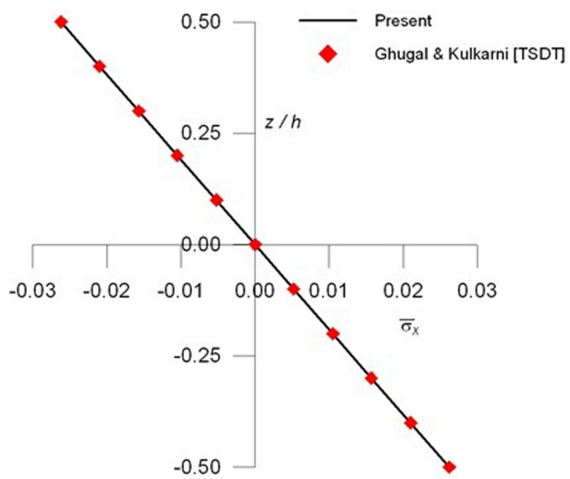

Fig. 2. Through thickness variation of in-plane normal stress $\bar{\sigma}_x$ for aspect ratio 10

Table 2. Comparison of transverse displacements $\bar{w}$ for the square isotropic plate subjected to uniformly distributed thermal load with respect to aspect ratio (S)

Source	$S = b/h$					
	2	4	5	10	20	100
Present (HYSDT)	0.1916	0.3831	0.4789	0.9579	1.9157	9.5787
Ref. [3]	0.1916	0.3831	0.4789	0.9579	1.9157	9.5787

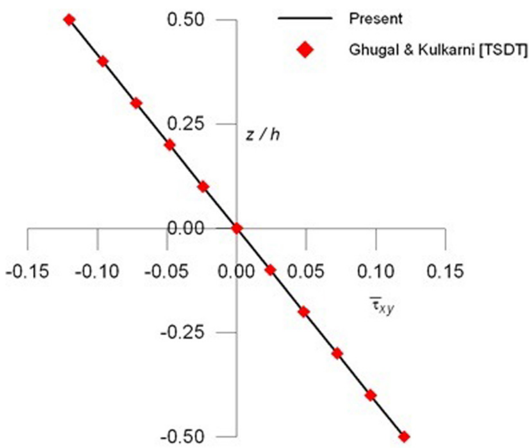

Fig. 3. Through thickness variation of in-plane shear stress $\bar{\tau}_{xy}$ for aspect ratio 10

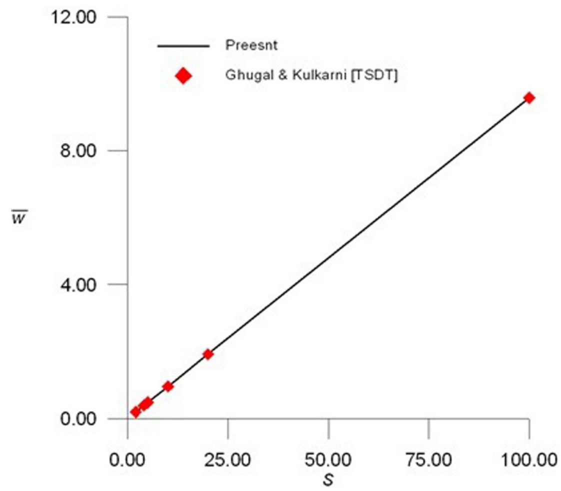

Fig. 4. Comparison of transverse displacement $\bar{w}$ with respect to aspect ratios

Table 3 and Fig. 5 show the comparison of transverse displacement for rectangular plates. The non-dimensional transverse displacements predicted by the presented theory are identical with those obtained by TSDT [3] for all (a/b) ratios. It is also observed from Table 3 and Fig. 5 that the values of transverse displacement increases with respect to increase in a/b ratios.

Table 3. Comparison of the transverse displacement $\bar{w}$ for the rectangular isotropic plates subjected to uniformly distributed thermal load

Source	Model	b/h	$a/b = 1$	$a/b = 1.5$	$a/b = 2.0$	$a/b = 2.5$	$a/b = 3.0$
Present	HYSDT	5	0.4789	0.6551	0.7403	0.7797	0.7977
Ref. [3]	TSDT	5	0.4789	0.6551	0.7403	0.7797	0.7977
Ref. [3]	FSDT	5	0.4789	0.6550	0.7403	0.7798	0.7977
Kirchhoff	CPT	5	0.4789	0.6600	0.7400	0.7800	0.8000
Present	HYSDT	10	0.9579	1.3102	1.4806	1.5593	1.5954
Ref. [3]	TSDT	10	0.9579	1.3102	1.4806	1.5593	1.5954
Ref. [3]	FSDT	10	0.9579	1.3095	1.4792	1.5582	1.5942
Kirchhoff	CPT	10	0.9579	1.3100	1.4804	1.5600	1.5951
Present	HYSDT	20	1.9157	2.6205	2.9612	3.1186	3.1907
Ref. [3]	TSDT	20	1.9157	2.6205	2.9612	3.1186	3.1907
Ref. [3]	FSDT	20	1.9157	2.6203	2.9611	3.1186	3.1907
Kirchhoff	CPT	20	1.9157	2.6200	2.9610	3.1200	3.1904

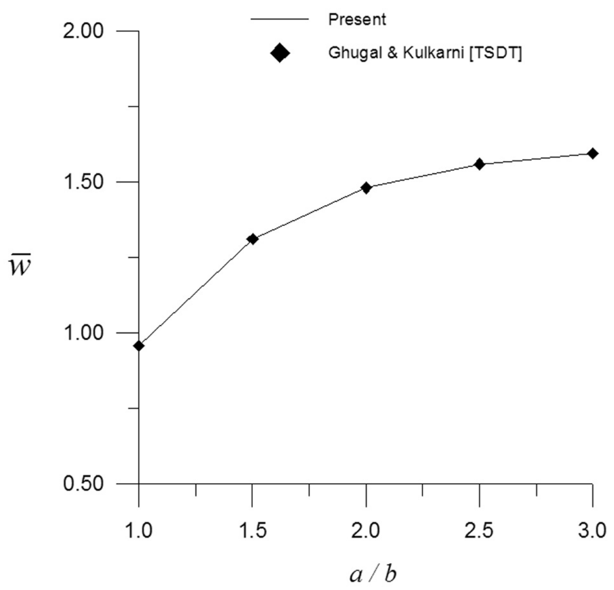

Fig. 5. Variation of maximum transverse displacement $\bar{w}$ with respect to a/b ratios for aspect ratio 10

7. Conclusions

In the present study, the hyperbolic shear deformation theory is used to investigate the bending response of isotropic plates under a uniformly distributed thermal load. Static solutions for simply supported isotropic rectangular plates are developed using the Navier procedure. From the numerical results and discussion, it is observed that, in case of isotropic plate, the displacements and in-plane stresses obtained by the presented theory are in excellent agreement with those of other refined theories whereas, it is also pointed out that, isotropic plate is subjected to zero transverse shear stresses due to thermal load.

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Appendix

The coefficients appearing in the governing differential equations and boundary conditions [Eq. (11–13)] are as follows:

$$D_{11} = D_{22} = \frac{AE}{(1-\mu^2)}, \qquad D_{12} = \frac{AE\mu}{(1-\mu^2)}, \qquad D_{66} = \frac{AE}{2(1+\mu)},$$

$$S_{11} = S_{22} = \frac{BE}{(1-\mu^2)}, \qquad S_{12} = \frac{BE\mu}{(1-\mu^2)}, \qquad S_{66} = \frac{BE}{2(1+\mu)},$$

$$SS_{11} = SS_{22} = \frac{CE}{(1-\mu^2)}, \qquad SS_{12} = \frac{CE\mu}{(1-\mu^2)}, \qquad SS_{66} = \frac{CE}{2(1+\mu)},$$

$$TD_{11} = \frac{AE}{(1-\mu^2)}\alpha_x, \qquad TD_{12} = \frac{AE\mu}{(1-\mu^2)}\alpha_x, \qquad TD_{22} = \frac{AE\mu}{(1-\mu^2)}\alpha_x,$$

$$TS_{11} = \frac{BE}{(1-\mu^2)}\alpha_x, \qquad TS_{12} = \frac{BE\mu}{(1-\mu^2)}\alpha_x, \qquad TS_{22} = \frac{BE\mu}{(1-\mu^2)}\alpha_x,$$

$$
\begin{aligned}
TTD_{11} &= \frac{AE}{(1-\mu^2)}\alpha_y, \qquad TTD_{12} = \frac{AE\mu}{(1-\mu^2)}\alpha_y, \qquad TTD_{22} = \frac{AE\mu}{(1-\mu^2)}\alpha_y,\\
TTS_{11} &= \frac{BE}{(1-\mu^2)}\alpha_y, \qquad TTS_{12} = \frac{BE\mu}{(1-\mu^2)}\alpha_y, \qquad TTS_{22} = \frac{BE\mu}{(1-\mu^2)}\alpha_y,\\
C_{55} &= \frac{DE}{2(1+\mu)}, \qquad C_{44} = \frac{DE}{2(1+\mu)},
\end{aligned}
\tag{A.1}
$$

where

$$
A = \int_{-h/2}^{h/2} z^2\,\mathrm{d}z,\; B = \int_{-h/2}^{h/2} zf(z)\,\mathrm{d}z,\; C = \int_{-h/2}^{h/2} f^2(z)\,\mathrm{d}z,\; D = \int_{-h/2}^{h/2} \left[\frac{\mathrm{d}f(z)}{\mathrm{d}z}\right]^2 \mathrm{d}z, \tag{A.2}
$$

where $f(z) = z\cosh\left(\frac{1}{2}\right) - h\sinh\left(\frac{z}{h}\right)$.