Homage to Professor Cyril Höschl

In 2015 Professor Cyril Höschl celebrated his 90th birthday. A researcher and university lecturer who has established himself as one of the leading figures of Czechoslovak and later on Czech mechanics. His curriculum vitae, with detailed descriptions of his family, personal and professional life, was recently published in the Bulletin of the Czech Society for Mechanics 1/2015.

Prof. Höschl, whose research beginnings can be traced back to machine failure diagnostics in the ČKD engineering company, is known not only as a recognised expert in the field of mechanics of materials, but also as a close collaborator of the Czech Society for Mechanics, which he co-founded in 1966. During his long life Professor Höschl was involved in the organisation of many research events and seminars, has acted as an editorial board member and reviewer of several journals. For his outstanding work has also been awarded with numerous distinctions such as the Golden Křižík's Medal and Josef Hlávka's Medal. He is an author or co-author of dozens of books, papers and educational texts, some of which, even after several decades, are still sought out by mechanical engineers from all generations.

Prof. Höschl was and still is famous for his pursuit to get straight to the core of problems and for his incessant quest to find the engineering truth. He is known for his intentions to show things in proper relations, as demonstrated by his many papers published, for example, in the Bulletin of the Czech Society for Mechanics, where he presented and solved difficult engineering problems. Using a light style language, but never departing from mathematical rigour in his papers, he was often able to present straight and surprising answers to perplexing scientific riddles. The best of these papers are collected in a recently issued book Höschl, C.: Essays on mechanics (ISBN 978-80-7372-455-9).

Editors believe that the following paper, written by M. Okrouhlík, the present chairman of the Czech Society of Mechanics, and titled *Achievements, agreements and quarrels of forefathers of mechanics*, that is dedicated to a part of history of mechanics, in which our 'mechanical' forefathers played important roles, was written in the spirit of Professor Höschl's intentions for seeing things in proper relations.

On behalf of the journal ACM
Jan Vimmr, Editor-in-Chief

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Achievements, agreements and quarrels of forefathers of mechanics

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Abstract

The presented paper is devoted to deeds of Galileo Galilei, Johannes Kepler, Robert Hooke, Christiaan Huygens and Isaac Newton with an intention to show their achievements in mechanics, their intellectual and scientific heritage, and also their personal vanities that sometimes led to harmful mutual relations.

Keywords: mechanics, history

1. Introduction

When the principles of mechanics and mathematics are introduced to undergraduate students, the known facts, laws and hypotheses are presented and employed as background for further subjects being taught in engineering curricula. The purpose of the paper is to show that the original procedures, leading to discovery, or rather to invention, of mechanical laws and principles, that we take for granted today, were lengthy, complicated and far from being straightforward. We try complementing them with contemporary mechanical and mathematical tools and teaching approaches.

Following lifetime destinies of a few forefathers of mechanics we try to unveil the difficulties and complications that they witnessed in derivations of their laws and formulas and to show how their personal grievances and bitter quarrels, that sometimes lasted for decades, complicated not only their mutual relations but in some case made obstacles in scientific communications between nations.

To the vast spectrum of scientists and to their achievements there is dedicated numerous, historically oriented literature — the author of this paper relied mainly on items listed in the References and mainly concentrated on lifes and works of Galileo Galilei, Johannes Kepler, Robert Hooke, Christiaan Huygens and Isaac Newton, trying to bring their narratives into proper relations.

2. Galileo Galilei (*15 February 1564, †8 January 1642)

He is usually referred to by his first name, i.e. Galileo, although his family name is Galilei.

The following famous phrase is attributed to him: *And yet it moves* or sometimes *albeit it does move*. In Italien it is: *Eppur si muove*. And in Czech: *A přece se točí*.

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He supposedly uttered those words at the Inquisition court in 1633, after being forced to take back his claims that the Earth moves around the Sun. However, the earliest biographers of Galileo do not mention that phrase and the records of the Inquisition trial do not cite it. The first account of the legend dates to a century after his death.

But there are always doubts. His presumed words, however in a slightly different spelling, i.e. *E pur si muove*, were found in 1911 on a painting by the Spanish painter Bartolomé Esteban Murillo. The painting had been completed within a year or two after Galileo died, (1643 or 1645). Today, it is believed that the painting is not historically correct, because it depicts Galileo in a dungeon. It is known, however, that after the trial Galileo was condemned, for the rest of his life, to the house arrest in the hills above Florence and not to jail.

Galileo discovered principle of inertia, stating that if an object has nothing acting on it, and is going at a constant velocity in a straight line, then it will go at the same velocity along the same line forever. Today, this principle is listed as the Newton's first law.

This is sometimes called Galilean invariance or Galilean relativity, which states that the laws of motion are the same in all inertial frames. It is also known under the name of Newtonian relativity. Galileo first described this principle using the example of a ship travelling at constant velocity. Any observer doing experiments below the deck would not be able to tell whether the ship was "smoothly" moving or stationary.

Galileo also discovered the isochrony of the pendulum — a principle important in design of clocks. He supposedly measured the period of a pendulum swings checking his own pulse.

3. Johannes Kepler (*27 December 1571, †15 November 1630)

He is best known for his laws of planetary motion, based on his works *Astronomia Nova, Harmonices Mundi*, and *Epitome of Copernican Astronomy*. These works provided one of the foundations for Isaac Newton's theory of universal gravitation.

Let's remind the laws that bear his name.

1. The orbits of planets are ellipses.

2. The areas, swept by focal radii of a planet in equal times (and in any part of the orbit) are equal.

3. The time the planet takes to go around the Sun is related to the size of orbit, more precisely to the square root of the cube of the size of the orbit, i.e. to the major axis of the ellipse.

Kepler's laws were not immediately accepted. Several major figures such as Galileo and René Descartes completely ignored Kepler's *Astronomia Nova*. Many astronomers, including Kepler's teacher, Michael Maestlin, objected to Kepler's introduction of physics into astronomy.

Final approval of his findings was culminated in Isaac Newton's *Principia Mathematica* (1687), in which Newton derived Kepler's laws of planetary motion from a force-based theory of universal gravitation.

4. Robert Hooke (*28 July 1635, †3 March 1703)

It is known that Hooke had a particularly keen eye, and was an adept mathematician and experimenter. In 1662 Hooke became a Curator of Experiments to the newly founded Royal

Society (established 1660) and took the responsibility for experiments performed at its weekly meetings. This position he held for over 40 years.

Hooke was appreciated for his inventiveness, remarkable experimental facility, and the capacity for hard work.

In 1663 and 1664, Hooke produced his microscopy observations, subsequently summarized in *Micrographia* in 1665.

Hooke's law of elasticity, in today's generalized form, is $\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl}$. Initially, in 1660, it was formulated for 1D linear variation of tension with extension in a linear spring. He described this discovery in the closely guarded anagram 'ceiiinosssttuv', whose solution he published later, in 1678, as *"Ut tensio, sic vis"*, meaning *As the extension, so the force*.

Hooke's work was also related to the development of the balance spring, or hairspring, which enabled to design a portable timepiece — a watch — to keep time with reasonable accuracy.

Since the torque generated by the coiled spring is proportional to the angle turned by the wheel, its oscillations have a nearly constant period.

The rivalry with Huygens

A bitter dispute between Robert Hooke and Christiaan Huygens on the priority of this invention continued for decades long after the death of both.

Hooke was also involved in the advances of the anchor escapement for pendulum clocks.

Hooke had lots of ideas and theories, and he wanted his personal credit for all of them. But the problem was that other scientists kept claiming they had come up with the same ideas long before he did.

The rivalry with Newton

One of the famous quarrels with Newton concerned the inverse square law of gravity. Hooke claimed the priority for its discovery and demanded that Newton acknowledge this in *Principia* which were just in the process of preparation. Newton reacted with bitter hostility and threatened to withdraw his manuscript from publication.

After all, it was Halley, who persuaded Newton to publish it. Newton however removed all the previous references to Hooke in his text.

However, the Royal Society declined to bear the cost of *Principia* publication, since its finances were exhausted, so Halley published Principia at his own expense.

Hooke supported the notion of gravity as a universal force, but according to Alexis Clairaut, a prominent French mathematician, astronomer and geophysicist of the eighteenth century, Hooke's articulation was more a basic idea than a full-fledged theory. Clairaut wrote … *what a distance there is between a truth that is glimpsed and a truth that is demonstrated*.

For thirty years, until his death in 1703, Hooke remained convinced that Newton would not have come up with inverse square law without his input and repeatedly accused Newton of appropriating theories that he himself originated.

Today, it is believed, that earliest statement about the inverse square law was found in *Astronomia Philolaica*, published in 1645 by a French mathematician and astronomer Ismael Boulliau.

From that time also comes the question raised by Halley, Hooke and Christopher Wren whether the assumption of an inverse square law would lead to Keplerian elliptical orbits. Newton has sent his proof to Halley, and later, it appeared in *Principia*.

There are rumors, indicating that the dislike was mutual, suggesting that later, when Newton became the President of the Royal Society, he tried to obscure Hooke's achievements, destroying the only known portrait of the man.

Much has been written about the unpleasant side of Hooke's personality, starting with comments by his first biographer, Richard Waller, who claimed that Hooke *was despicable and melancholy, mistrustful, and jealous person*.

On internet one can find a truly tabloid item bearing the title: *Was Robert Hooke really the greatest asshole in the history of science?* A careful reader might find, however, that the contents of this entry are much more reasonable than the horribly sounding title promises.

5. Christiaan Huygens (*14 April 1629, †8 July 1695)

Huygens is known for collision formulae, pendulum clock, wave theory, musical tuning, etc.

Leibniz was tutored in mathematics by Huygens. In extensive correspondence with Leibniz, Huygens showed reluctance to accept the advantages of infinitesimal calculus.

Huygens sought his own explanation of the force of gravity that would avoid the action at a distance.

Huygens designed more precise clocks that were available at that time. The formula for the period of pendulum swing, i.e. $T = 2\pi\sqrt{\frac{l}{g}}$, was derived by Huygens. The oldest known Huygens-style pendulum clock is dated 1657. Huygens also developed a spiral balance spring watch — independently of Robert Hooke. The controversy over the priority, however, persisted over the decades.

Huygens believed into the constant velocity of light even before the experimental confirmation by a Danish astronomer Olaus Roemer.

Among other things Huygens, in his paper devoted to the suspension bridge, demonstrated that the catenary is not a parabola. A catenary is the curve that an idealized hanging cable assumes under its own weight when supported only at its ends. Today we take for granted that the catenary equation is $y = a\cosh(x/a)$ or $y = \frac{a}{2}\left(\mathrm{e}^{\frac{x}{a}} + \mathrm{e}^{-\frac{x}{a}}\right)$.

In 1678, Huygens proposed that every point which a luminous disturbance reaches becomes a source of a spherical wave; the sum of these secondary waves determines the form of the wave at any subsequent time.

In solid continuum mechanics, when halfspace is being loaded by a sudden point force, we have two kinds of waves which might propagate, i.e. longitudinal and transversal. To each point being hit in this area becomes a source of both types of waves. The straight line (von Schmidt wavefront) is the envelope of secondary longitudinal waves. See Fig. 1.

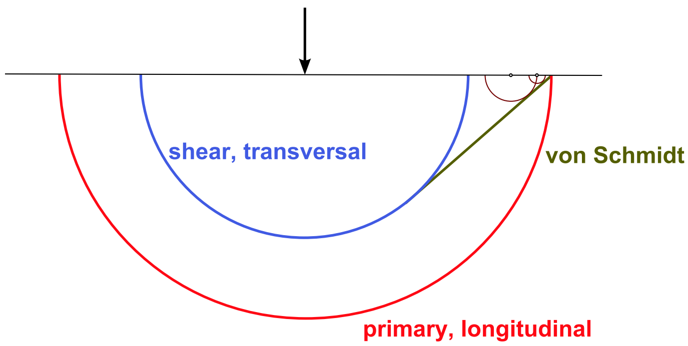

Fig. 1. 2D wavefronts in solid elastic isotropic half-space

6. Gottfried Wilhelm Leibniz (*1 July 1646, †14 November 1716)

In philosophy Leibniz is known for his optimism. He stated that our Universe is the best possible one that God could have created or that God always chooses the best.

Compare his views to variational principles. Equilibrium and the optimum path correspond to minimum energy considerations — which is evidently the 'best' — at least from an engineering point of view.

His idea was often lampooned by others especially by Voltaire.

Leibniz approached one of the central criticisms of Christian theism: If God is all good, all wise and all powerful, how did evil come into the world? The answer (according to Leibniz) is that, while God is indeed unlimited in wisdom and power, but his human creations are limited both in their wisdom and in their will (power to act). This predisposes humans to false beliefs, wrong decisions and ineffective actions in the exercise of their free will.

Leibniz claims that there must always be a sufficient reason for anything to exist, for any event to occur.

Leibniz belongs to the most important logicians from Aristotle to George Boole and Augustus de Morgan. Leibniz enunciated the principal tools of logic which are known today as conjunction, disjunction, negation, exclusion, etc. See Fig. 2.

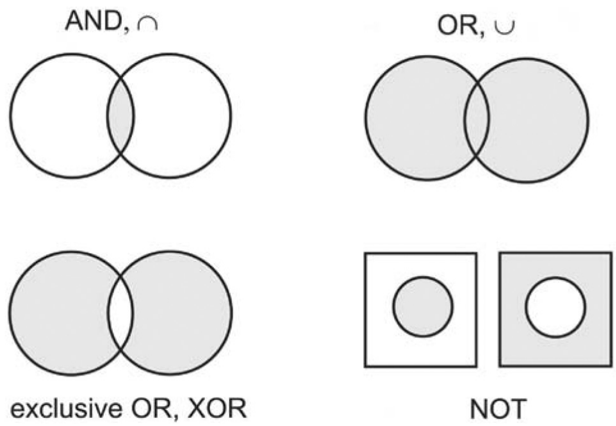

Fig. 2. Logical operations

Leibniz was the first who saw that the coefficients of a system of algebraic equations could be arranged into an array, which is now called a matrix.

Leibniz is credited — together with Isaac Newton — with discovery of infinitesimal calculus.

The first account of calculus was published by Leibniz in 1684 under the title *Nova Methodus pro Maximis et Minimis, itemque Tangentibus, qua nec Irrationles Quantitates Moratur — A new method for maxima and minima and also for tangents which is not obstructed by irrational quantities*.

Here, one can find formulas having the today's appearance as

$$\mathrm{d}(xy) = x\,\mathrm{d}y + y\,\mathrm{d}x, \qquad \mathrm{d}\left(x/y\right) = \left(y\,\mathrm{d}x - x\,\mathrm{d}y\right)/y^2, \qquad \mathrm{d}x^n = nx^{n-1}.$$

From 1711 until his death, Leibniz was engaged in a bitter dispute with Newton, over whether he invented calculus independently of Newton. See [1].

Leibniz defines *vis viva* (Latin for *living force*) as mv^2, twice the today's kinetic energy. He claimed that, under certain circumstances, the kinetic energy is conserved.

7. Isaac Newton

According to the English calendar which was ten days out of step with the calendar of most European countries, was born on Christmas Day 1642. The corresponding day in Europe was 4 January 1643. So the frequently appearing statement that Newton was born the same year Galileo died (i.e. on 8 January 1642) is at least questionable. Newton died on 31 March 1727.

Young Newton became acquainted with works of Gaileo, Fermat, Huygens and others. In a letter to Robert Hooke, he wrote a sentence, which became famous: *If I have seen further, it is because I have stood on the shoulders of giants*.

Today, Newton is appreciated for four major discoveries.

1. The binomial theorem.
2. The calculus.
3. The law of gravitation.
4. The nature of colors.

Newton's masterpiece, titled *Philosophiae Naturalis Principia Mathematica*, i.e. *Mathematical Principles of Natural Philosophy*, was published in 1687. *Principia*, for short, are composed of three books related to laws of motion, law of universal gravitation and derivation of Kepler's laws of planetary motion.

Even long after the first appearance of *Principia*, Newton's ideas had met with considerable opposition. Eminent mathematicians of seventeenth century, as Huygens, Leibniz, John Bernoulli, Cassini and others, strongly disagreed with Newton's theory of gravitation and with notion of inertia.

Mainly, it was the invisible *gravitational action at a distance* that was difficult to accept.

Newton's opponents claimed that if one says that things fell because of gravity — then the mystery is merely given a name.

His learned colleagues claimed: "Gravity — it does not mean anything — it tells us nothing about why." And Newton supposedly replied: "It tells you how it moves, not why."

One of Newton's frequently cited statement is: *Hypotheses non fingo — I contrive no hypotheses.* See [4].

Here, one can pose for a moment and remind a good-humored remark related to origins of inertia, which is attributed to Feynman's father. See [4].

Richard Feynman, as a little boy, pulled the toy wagon with a ball inside and observing the motion of the ball he approached his father pondering:

When I pull the wagon, the ball rolls to the back of the wagon.

When I suddenly stop, the ball rolls forward.

And asking:

Why is that?

And his father, being a tailor by profession, answered. *Things that are moving try to keep on moving and things that are standing still tend to stand still. This tendency is called inertia but nobody knows why it is true.*

This is an excellent and amusing story, which, however, almost to the word paraphrases the Newton's first law. A reader might have a feeling that it was invented by Richard Feynman long after he has grown up and has been awarded the Nobel Prize.

Back to *Principia*. Newton's text is written in Latin and his mathematical lemmas and proofs are presented by means of medieval geometry.

English translation of *Principia*, together with the detailed explanation of Newton's text and his procedures and geometrical proofs can be found in Guicciardini book [7].

T. D. Whiteside [13], for the benefit of modern readers, translates Newton's demonstrations into modern mathematical language. Guicciardini claims, however, that doing this Whiteside obscures and suppresses the originality and diversity of Newton's procedures.

Newton's *Principia* first appeared in 1687, two further editions, in 1713 and 1726. But the calculus methods are not covered and practically not used in *Principia*. Instead, Newton provided proofs of his statements using the principles of classical Greek geometry.

Newton was so sensitive to criticism that after attacks from Hooke and others on his paper concerning the nature of color, he determined to publish nothing further. For fifteen years he really published nothing until Halley urges him to publish *Principia*, which, at that time, has already been fully completed.

Newton's notation in Principia

In Newton's text, see [7], the symbol $\propto$ is used to represent the statement 'is proportional to'.

When Newton writes ABq or $ABquad$ he actually means $\left(\overline{AB}\right)^2$, where $\overline{AB}$ is the length of the line between points A and B. Similarly, $ABcub$ is understood to be $\left(\overline{AB}\right)^3$. Newton's statement '$A$ is as B directly' means that A is proportional to B. Similarly, 'A is as B inversely' means that A is proportional to $1/B$. And his statement *'force is the square of velocity'* should be translated as the force is proportional to the square of velocity.

In *Principia* there is no explicit occurrence of *mass* in formulas related to central force motion, resisted motions, etc. This is due to the fact that in those days the mathematicians thought in terms of proportions. So the factors (constants of proportionality) are not made explicit and thus one cannot rely on the dimensional analysis when checking Newton's formulas. Often, force is equated with acceleration. Original statement of the Newton's second law, as translated from Latin, [7], has the form:

> *The change of motion is proportional to the motive force impressed and is made in the direction of the straight line in which that force is impressed.*

That statement should be understood in such a way that the change of velocity is proportional to the force and is in the same direction as the applied force. The vectorial character of the statements is expressed in words, but the mass is not mentioned.

What is worth mentioning is that the famous formula, known to today's college and undergraduate students, i.e. $\vec{F} = m\vec{a}$, is not — at least in this form — found in *Principia*.

Even the recent English translation of Principia by Cohen and Whitman do not represent an easy bedside reading. Take for example the Lemma 10 of Book 1, which states:

> *The spaces, which a body describes when urged by any finite force — whether that force is determinate and immutable or is continually increased or continually decreased — are, at the very beginning of the motion, in the duplicate ratio of times.*

This should be understood as the statement that *the distance travelled from the rest is proportional to the square of time*.

Our undergraduate students, considering $F = \text{const.}$, nonzero starting displacement, together with nonzero initial velocity, might proceed as follows

$$ma = F, \quad m\frac{\mathrm{d}v}{\mathrm{d}t} = F, \quad \int_{v_0}^{v} \mathrm{d}v = \frac{F}{m}\int_0^t \mathrm{d}t, \quad v = v_0 + \frac{F}{m}t, \quad \frac{\mathrm{d}s}{\mathrm{d}t} = \left(v_0 + \frac{Ft}{m}\right),$$

$$\int_{s_0}^{s} \mathrm{d}s = \int_0^t \left(v_0 + \frac{Ft}{m} \right) \mathrm{d}t \Rightarrow s - s_0 - v_0 t = \frac{F}{2m} t^2 \qquad \text{or} \qquad s = s_0 + v_0 t + \frac{1}{2} a t^2.$$

Our students are coming to the same conclusion as Newton, namely that the distance travelled from the rest is proportional to the square of time or by other words that the displacement is proportional to acceleration.

During the winter of 1664–1665 Newton established his first mathematical discovery — binomial theorem. For instance he found

$$\left(1 - x^2\right)^{\frac{1}{2}} = 1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 - \frac{1}{16}x^6 - \frac{1}{128}x^8 \ldots \qquad \text{or} \qquad (1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 \ldots$$

Today's mathematicians say that Newton had a rather intuitive concept of the convergence — he thought that binomial series can be safely applied when the argument x is small. But is not that a feeling of the most of today's mechanical engineering community?

Concept of infinitesimal quantities, as defined and used in the seventeenth century, was based on the so-called principle of cancelation that stated the following. If α is infinitesimally small and A is finite then $A + \alpha = A$.

Compare this with *unit round-off error* or *machine epsilon* defined in computer science, which is, however, the **finite** distance from 1.0 to the next larger double precision number.

```
% machep.m
clear; format long e
ep = 1; i = 0;
while 1 + ep > 1,
  ep = ep / 2; i = i + 1;
end
[ep i eps]'
```

Matlab, with standard representation of real numbers, designating 53 bits to mantissa, gives the `ep` value, number of mantisa bits and the value of unit round-off error as follows

```
1.110223024625157e-016 5.300000000000000e+001 2.220446049250313e-016.
```

Fifteen years after the first edition of *Principia* appeared, and after the death of Robert Hooke, Newton's aversion to publication died away and he, in 1704, finally published his *Optiks*, to which two mathematical appendices were attached. One of them, *De Quadratura Curvarum*, contained the intelligible account of Newton's calculus procedures.

Newton's terminology concerning fluents and fluxions

- Fluents are quantities continuously changing in time, as lengths, areas, volumes, etc.
- Fluxions are rates of change of continuously changing quantities, i.e. of fluents.

In Newton's words: *Fluxions of quantities are in the first the ratio of their nascent parts or, what is exactly the same, in the ultimate ratio of those parts, as they vanish.*

The words *prime and ultimate ratios* correspond to *rationes primae et ultimae* in the original Latin text.

Translated into modern language: Infinitesimal quantities, which — in the process of coming into existence form nothing, or vanishing into nothing — pass through a state in which they are neither finite nor nothing. Modern mathematics circumnavigates this rather complicated and cumbersome statement by the well established limit approach.

Newton in his texts regards the symbol o as a very small interval of time, i.e. Δt in our notation. For variables x, y the rate of their change (in time) is indicated by op, oq — meaning $\Delta x, \Delta y$. The ratio q/p thus corresponds to $\Delta y/\Delta x$, i.e. to the slope of the curve $y = f(x)$. In his later texts Newton replaced the quantities p, q by dotted letters, i.e. by $\dot{x}, \dot{y}$.

Newton discovery preceded that of Leibniz by about ten years, but the discovery of Leibniz was independent of that of Newton.

Leibniz is entitled to priority of publication. He published his findings in 1684 in *Nova Methodus* mentioned above.

Leibniz arrived at the same conclusions as Newton but his approach was more general. It could have been applied to any function (rational, irrational, algebraic or transcendental).

It should be noted that Leibniz established the notation style, which is used up to the present time.

Thus $\mathrm{d}x, \mathrm{d}y$ are the smallest possible differences of x, y. For the sum of ordinates under a curve he wrote $\int y\,\mathrm{d}x$, the today's integral operator being originally an enlarged letter S, indicating the Latin word summa. These findings were published 1686 in Leibniz's paper *On Recondite*[1] *Geometry and the Analysis of Indivisibles and Infinities*[2]. However, the words *integral* was not used by Leibniz at that time — instead he named the procedure by the term *recondite geometry*. The term integral was first used in a paper published by Bernoulli brothers in 1690. The term *integral calculus* appeared later, in 1690, in a joint paper written by Johann Bernoulli and Leibniz. See [1].

Newton's analytical method of fluxions could be considered as the counterpart of Leibnizian differential and integral calculus. Both approaches are analogous but not identical.

Three short book reviews are presented at the end of this short essay. The books are *The Calculus Wars* by Jason Bardi [1], *Newton: The Making of Genius* by Patricia Fara [3] and *Feynman's lost lecture* by Goodstein, David L. and Goodstein, Judith R. [6].

The Calculus Wars by Jason Bardi [1].

Into the greatest details the author describes a bitter fight, between Gottfried Wilhelm Leibniz (1646–1716) and Sir Isaac Newton (1642–1726), concerning the priority of invention of calculus.

Newton invented calculus, which he called method of fluxions and fluents during his most creative years of 1665 and 1666, but kept his work secret for most of his life. It was not until 1703 when his *Optiks*, together with the appendix titled *Tractatus de Quadratura Curvarum* (*On the Quadrature of Curves*), was published. This treatise, the original form of which was actually written back in 1691, presents the basics of the method of fluxions and fluents. So, the official date of Newton's calculus publication is 1703.

Leibniz came upon calculus later, between 1672 and 1676, and published his findings in two papers that appeared in 1684 and 1686.

Today, both Leibniz and Newton are regarded as independent inventors.

The battle lasted for more than ten years. Newton and Leibniz attacked each other both openly and in secret. Newton continued publishing defenses of himself long after Leibniz's death accusing him of plagiarism.

[1]Recondite means hidden, unclear.

[2]Full text in Latin together with English translation is at www.17centurymaths.com/contents/Leibniz/ae19.pdf

Leibniz was fighting back publishing anonymously treatise suggesting that Newton had borrowed ideas from him.

This excellently written book has qualities fully satisfying both historically and 'mechanically' oriented readers.

The Making of Genius by Patricia Fara [3].

Patricia Fara presents quite unorthodox views of Isaac Newton and claims:

> *Isaac Newton is now universally celebrated as a scientific genius. Yet Newton himself was not a scientist. The word scientist was not even invented more than 100 years after his death.*

Very often, Patricia Fara, instead of appreciating what Newton has done, ponders about meanings of the many words, namely scientist and genius.

Today, and even for a few previous centuries, it is generally believed that what Newton has achieved is a pure essence of scientific pursuit and his life long activities document the proper meaning of the word science. Isaac Newton succeeded in explaining the nature of things utilizing mathematics and proving his results with experimental findings. He, in the preface of the *Principia*, writes: . . . *rational mechanics is the science of motions resulting from any force whatsoever. . .*

As far as the definition of the word genius is concerned we might use the Immanuel Kant's statement that . . . *the genius is a man not only of wide range of mind but also of great intellectual greatness, who is epoch-making in everything he undertakes.*

Kant also stated that the *genius is a talent for producing an original piece of work that operates completely independently of any rules*.

This way, the Patricia Fara forces the reader to mentally discuss semantic contents of words distracting him from what his mechanically oriented mind is really interested in.

Apparently, Patricia sees things in a completely different light:

> *Newton deliberately made the book accessible only to privileged knowledgeable elite to be understood by able Mathematicians.*

So, in her view the mathematical language is an obstacle to a clear understanding. Compare it with that of Richard Feynman. He was often asked to explain physical laws in words instead of symbols. Feynman claims [4] that this is not fully possible since mathematics is not just a language. *Mathematics is the language plus reasoning*.

So Patricia, instead of dealing with Newton's achievements, inventions and discoveries, is interested in *question-marks hanging over his life* as:

- did he experience a period of insanity,
- did he enjoy homosexual relations with younger men or,
- was he emotionally damaged by his father's death before he born?

As far as Newton's homosexuality is concerned there are recorded Newton's own words, supposedly proclaimed at the end of his life, that he remained a virgin during his lifetime.

Patricia's comment to the famous statement about the shoulders of giants[3] is as follows: Hooke was commonly described as very short, even hunchbacked, and here is a theory that

[3]Newton, in one of the letters addressed to Hooke, declared: *If I have seen further it is by standing on the shoulders of giants.*

Newton's mention of 'giants' was his way of saying Hooke had no influence on his work, implicating that Hooke was no giant at all.

So, even if the Fara's book contains a lot of interesting personal and historical details of Newton's era, Newton himself and of his contemporaries, the reviewer cannot get free of the feeling that Patricia tries to discredit Newton's geniality. Evidently, the book is intended for a different, non-mechanical oriented audience.

Feynman's lost lecture by David Goodstein and Judith Goodstein [6].

The book not only describes the detective story how the Feynman's lecture, initially titled *The Motion of Planets around the Sun*, was lost, found in pieces and completely reassembled and restored later, but it primarily depicts the Feynman's unorthodox approach to Isaac Newton's geometric demonstration of the law of ellipsis in the *Principia*.

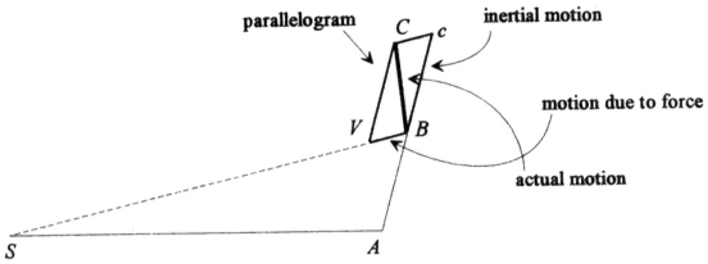

Fig. 3. Motion of a body in gravitational field (from [6])

The Newton's Proposition 1 of Book 1 reads:

> *The areas which bodies made to move in orbits describe by radii drawn to an unmoving centre of forces lie in unmoving planes and are proportional to the times.*

Feynman in his lecture follows the reasoning presented in Newton's *Principia*. The planet (mass particle) with no force acting on it would proceed in the straight line to the point c. See Fig. 3. Any kind of centripetal force aiming to S, the Sun, evokes the displacement BV. The resulting motion is given by a diagonal of the indicated parallelogram. This step, repeated with equidistant time intervals, leads to the complete elliptical orbit. Notice that mass plays no role.

Feynman explains the Newton reasoning in today's terminology. In his own words:

> *We have used the Newton's first law (the law of inertia), Newton's second law (any change of motion is in the direction of the impressed force) and the idea that the considered force is centripetal, i.e. is directed toward the Sun. Nothing else. . . . So any other kind of force would have produced the same result, provided only that the force is directed towards the Sun.*

Later in his lecture Feynman shows how Newton deduced the inverse-square-of-the distance-nature of gravity from the Kepler's third law. Feynman's proofs are quite lengthy, they are described in 107 pages of [6] and are accompanied by tens of sketches. Even if each individual step in his lecture is elementary, the proof taken as a whole is far from being simple.

Today's undergraduate students, using their contemporary knowledge and tools, would proceed when asked to determine the motion of a particle in the force field with a central attractive force, being proportional to the distance, might proceed as follows. Let m is the mass

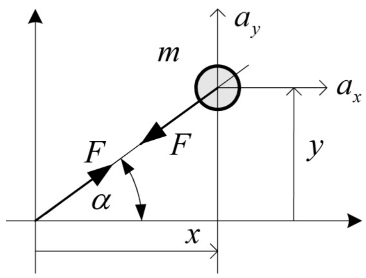

Fig. 4. A particle in the force field with a central attractive force, proportional to the distance

of the particle, c is the proportionality constant and the initial conditions are $t = 0$, $x = x_0$, $y = 0$, $v_y = v_{y0}$. See Fig. 4.

It should be emphasized that in this case we do not intend solving a motion of a particle in the gravity field, where the gravity force would be proportional to the inverse of the distance squared.

Thus the equations of motion of the particle are $ma_x = -F\cos\alpha$ and $ma_y = -F\sin\alpha$. Denoting c the coefficient of proportionality, the 'force of attraction' is $F = c\sqrt{x^2+y^2}$. Realizing that $\cos\alpha = \frac{x}{\sqrt{x^2+y^2}}$, $\sin\alpha = \frac{y}{\sqrt{x^2+y^2}}$ and substituting into equations of motion we obtain $\ddot{x} + \frac{c}{m}x = 0$, $\ddot{y} + \frac{c}{m}y = 0$, i.e. two ordinary differential equations of the second order with constant coefficients. Considering the initial conditions in the form $t = 0$, $x = x_0$, $y = 0$, $v_x = 0$, $v_y = v_{y0}$ and introducing a new variable $c/m = \Omega^2$, the students follow the familiar route leading to $x = A\sin(\Omega t + \gamma_1)$, $y = B\sin(\Omega t + \gamma_2)$. It is obvious that the trajectory of the particle, after four unknown constants from initial conditions are determined, is elliptic. Parametric equations of that ellipse are $x = x_0\cos\Omega t$, $y = \frac{v_{y0}}{\Omega}\sin\Omega t$. Notice that the trajectory of the particle does not depend on its mass. The revolution period is $T = \frac{2\pi}{\Omega}$. Animated picture of the orbit could simply be provided by

```
% edu_UL_2013_DY_02_02
clear
x0 = 4; v0 = 20; omega = 10; T = 2*pi/omega; t_range = 0:pi/360:T;
x1 = x0*cos(omega*t_range); y1 = v0*sin(omega*t_range)/omega;
xmax = 1.1*max(x1);
ymax = 1.1*max(y1);

figure(1)
plot(x1,y1,'k-');
axis([-xmax xmax -ymax ymax]);
hold on
for t = t_range
 x = x0*cos(omega*t);
 y = v0*sin(omega*t)/omega;
 plot(x,y,'or', 'linewidth', 2);
 pause(0.1)
end
hold off
print -djpeg -r300 fig_DY_02_02_02
% end
```

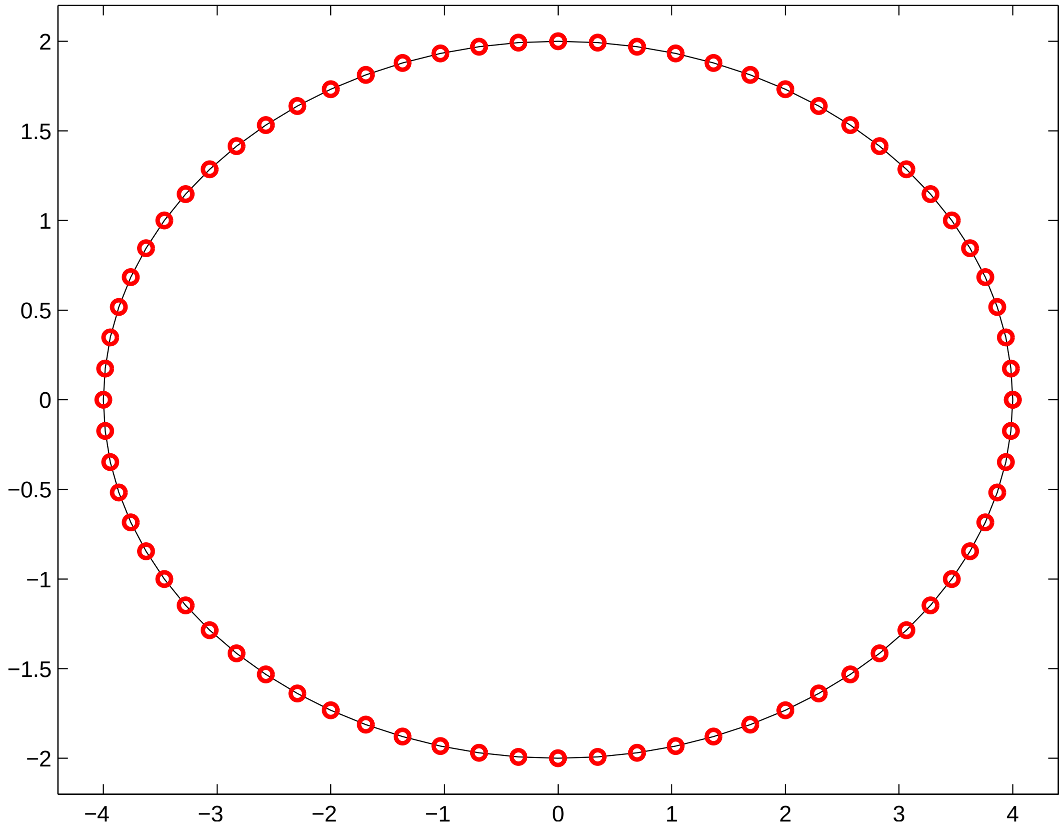

Fig. 5. Motion of a particle in the force field with a central attractive force, proportional to the distance

So, even if the centripetal attraction force is assumed to be proportional to the distance, the orbit remains elliptical.

8. Conclusions

The paper tries shedding light on the historical background of mechanics with the intention to present personal relations between our forefathers. Also, it implies how difficult and non-straightforward was the way to contemporary tools being used in mechanics on everyday's basis. The author believes that the paper might be of interest to graduate students starting their dynamics curriculum. The references [2, 5, 8–12] are recommended for further study.

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