Applied and Computational Mechanics 3 (2009) 259–266

Identification of low cycle fatigue parameters

M. Balda[a,*]

[a] ŠKODA VÝZKUM, s.r.o, Tylova 57, 316 00 Plzeň, Czech Republic

Received 7 September 2009; received in revised form 18 December 2009

Abstract

The article describes a new approach to the processing of experimental data coming from low-cycle fatigue (LCF) tests. The data may be either tables from the standard tests, or a time series of loading processes and corresponding numbers of cycles to damage. A new method and a program for the evaluation of material parameters governing the material behavior under a low cycle loading have been developed. They exploit a minimization procedure for an appropriate criterion function based on differences of measured and evaluated damages.
© 2009 University of West Bohemia. All rights reserved.

Keywords: low cycle fatigue, measurements, data processing, material parameters

1. Introduction

Estimation of fatigue lives of structures and their parts is rather a delicate task. Usually it is not very easy to obtain reliable data for the material of a part to be evaluated. Experiments, performed for getting material fatigue data which are necessary for the evaluation of required parameters, are time consuming and in consequence of it also rather expensive. No wonder that fatigue data of a particular material are not found very often in literature. Moreover, every material has rather wide tolerance bands for property values, fatigue ones included. As a result, the parameter values found in the literature may be far from the real ones.

There are some empirical formulae available to facilitate works of designers on fatigue estimates [5] in the literature. The formulae are based on the knowledge of Young's modulus E and strength R_m of the material, which are easily obtained from a general tensile test. However, values of such parameters give only an approximate information on fatigue lives, which may be by orders far from the actual ones. Hence, experimental data should be at disposal for a reliable fatigue life evaluation.

All general methods for estimating of fatigue lives of structures exposed to the low cycle fatigue loading are based on three equations. The first of them has been published by Basquin [4] in 1910:

$$\sigma_a = \sigma_f^{'} \left(2\,N_f\right)^b \,. \tag{1}$$

It expresses the exponential relationship between a number of cycles to failure N_f and a stress amplitude σ_a. Similarly, it holds between the amplitude of plastic strain ε_{ap} and a corresponding fatigue life N_f:

$$\varepsilon_{ap} = \varepsilon_f' \left(2\,N_f\right)^c \,. \tag{2}$$

The equation is named after two scientists, Coffin and Manson, who independently found it in the fiftieth of the past century (see [6] and [8], respectively).

*Corresponding author. Tel.: +420 379 852 280, e-mail: miroslav@balda.cz.

The third equation describes a total strain as a sum of elastic and plastic strains

$$\varepsilon_t = \varepsilon_e + \varepsilon_p = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{\frac{1}{n'}} , \tag{3}$$

which plotted creates the cyclic stress-strain curve. This curve forms a hysteresis loop in a single fatigue cycle. The area of that loop is the dissipated energy per unit volume during a cycle.

The above written equations contain seven material parameters besides amplitudes of a stress σ_a, strain ε_{at} and ε_{ap} and a fatigue life N_f:

σ'_a	fatigue strength coefficient,	b	fatigue strength exponent,
ε'_f	fatigue ductility coefficient,	c	fatigue ductility exponent,
n'	cyclic strain hardening exponent,	E	Young's modulus.
K'	cyclic strength coefficient,		

Only first four parameters are independent, because n' and K' are functions of the remaining ones:

$$n' = \frac{b}{c} \qquad \text{and} \qquad K' = \frac{\sigma'_f}{\varepsilon'^{\,n'}_f} . \tag{4}$$

All parameters should be obtained by a processing of experimental data. The fundamental guide for low cycle fatigue tests is the book [1], where everything concerning specimens, testing machines, and ways of classical testing is described. Classical tests are performed on sets of $m \geq 10$ specimens. Since such tests are too time consuming, new testing procedures have been searched. Of course, that the processing of experimental data is procedure dependent.

2. Processing of classical LCF test data

Experimental data of classical LCF tests are generally put into table $\boldsymbol{X}$ built out of column vectors

- $\boldsymbol{k}$ specimen identifiers (say, specimens numbers)
- $\boldsymbol{\sigma}_a$ stress amplitude
- $\boldsymbol{\varepsilon}_t$ total strain amplitude
- $\boldsymbol{\varepsilon}_p$ plastic strain amplitude
- $\boldsymbol{\varepsilon}_e$ elastic strain amplitude
- $\boldsymbol{N}_f$ Number of cycles to a failure – fatigue life

The minimum table for the processing contains at least the columns $\boldsymbol{\sigma_a}$, $\boldsymbol{\varepsilon_t}$ and $\boldsymbol{N_f}$, provided Young's modulus E be known from a tensile test. In such a case, $\boldsymbol{\varepsilon_p}$ may be evaluated as $\boldsymbol{\varepsilon_p} = \boldsymbol{\varepsilon_t} - \boldsymbol{\sigma_a}/E$. The unknown LCF material parameters are sought by linear regression in logarithmic axes.

Since σ_a and ε_{ap} under equations (1) and (2) are exponential functions, they become straight lines when plotted in log-log papers, because

$$\log_{10}(\sigma_a) = \log_{10}(\sigma_f^{'}) + b\log_{10}(2N_f) \tag{5}$$

$$\log_{10}(\varepsilon)_{ap} = \log_{10}(\varepsilon'_f) + c\log_{10}(2N_f) . \tag{6}$$

Both equations may be gathered into the matrix form

$$\underbrace{\left[\ \boldsymbol{j}\ ,\ \log_{10}(2\boldsymbol{N_f})\ \right]}_{\boldsymbol{A}}\ \underbrace{\left[\begin{array}{cc} \log_{10}(\sigma_f^{'})\ , & \log_{10}(\varepsilon_f')\\ b\quad , & c\end{array}\right]}_{\boldsymbol{X}} = \underbrace{\left[\ \log_{10}(\boldsymbol{\sigma_a})\ ,\ \log_{10}(\boldsymbol{\varepsilon_{ap}})\ \right]}_{\boldsymbol{B}}, \tag{7}$$

where $\boldsymbol{j}$ is a column vector of all ones. The matrix $\boldsymbol{X}$ of unknowns can be evaluated by means of the Moore-Penrose pseudoinverse matrix $\boldsymbol{A}^+$ of $\boldsymbol{A}$ in the form:

$$\boldsymbol{X}\ =\ \boldsymbol{A}^+\boldsymbol{B}\qquad \text{or}\qquad \boldsymbol{X}\ = (\boldsymbol{W}\,\boldsymbol{A})^+\,\boldsymbol{W}\,\boldsymbol{B}\,, \tag{8}$$

provided a weighing of measured data by the diagonal matrix $\boldsymbol{W}$ were applied.

3. Estimation of fatigue life

Plenty of formulae for evaluating fatigue life N_f have been derived from the Basquin and Manson-Coffin equations (see (1) and (2)). The most simple of them is that by **Crews and Hardrath** which expresses the number of cycles of fixed amplitudes to damage in the closed form:

$$N_f = \frac{1}{2}\left(\frac{\sigma_a}{\sigma_f^{'}}\right)^{\frac{1}{b}} \tag{9}$$

Landgraf proposed how to include the mean stress σ_m into the fatigue life estimation by reducing $\sigma_f^{'}$ for those cycles in which a crack opens.

$$N_f = \frac{1}{2}\left(\frac{\sigma_a}{\sigma_f^{'} - \sigma_m}\right)^{\frac{1}{b}} \tag{10}$$

Morrow derived a nonlinear formula for N_f from the equivalence of strains defined by the equation (3):

$$\varepsilon_f'\,(2\,N_f)^c + \frac{\sigma_f^{'}}{E}\,(2\,N_f)^b - \varepsilon_{at} = 0 \tag{11}$$

Morrow-Landgraf formula respects the influence of a mean stress of a cycle in the same way as in the Landgraf's method:

$$\varepsilon_f'\,(2\,N_f)^c + \frac{\sigma_f^{'} - \sigma_m}{E}\,(2\,N_f)^b - \varepsilon_{at} = 0 \tag{12}$$

Topper brought an idea of the equivalence of stress-strain combination. It may be easily expressed by the equation (3) multiplied by σ_a. After the substitution for its elements, the following nonlinear equation is generated:

$$\frac{\sigma_f^{'2}}{E}\,(2\,N_f)^{2\,b} + \sigma_f^{'}\,\varepsilon_f'\,(2\,N_f)^{b+c} - \varepsilon_{at}\,\sigma_a = 0 \tag{13}$$

SWT is an abbreviation of the names of authors **Smith, Wetzel and Topper**, who introduced an effect od the mean stress into the Topper's formula by changing σ_a by the peak stress $\sigma_h = \sigma_a + \sigma_m$. It has the following form:

$$\frac{\sigma_f^{'2}}{E}\,(2\,N_f)^{2\,b} + \sigma_f^{'}\,\varepsilon_f'\,(2\,N_f)^{b+c} - \varepsilon_{at}\,(\sigma_a + \sigma_m) = 0 \tag{14}$$

In the recent time, the author of this contribution has presented a new speculative way of respecting a mean stress in fatigue life computations [2]. It may be demonstrated on the simplest case of two equations due to Crews & Hardrath (9), and Landgraf (10). The equation (10) holds for the raising reversal and (9) for the other. After summing both modified equations, one gets

$$2\sigma_a = (2\sigma_f^{'} - \sigma_m)\,(2N_f)^b. \tag{15}$$

This is the reason why $\sigma_m/2$ occurs in the above presented formulae instead of σ_m in his modifications. All formulae for the evaluation of fatigue life have some mutually common properties, which enable to generalize them into three groups gathered in tab. 1.

Table 1. A survey of formulae for fatigue life evaluation

Method	Formula
Crews & Hardrath Landgraf Balda 1	$N_f \;=\; \dfrac{1}{2}\left(\dfrac{\sigma_a}{\sigma_f^{'} - k_m\,\sigma_m}\right)^{\frac{1}{b}}$
Morrow Morrow & Landgraf Balda 2	$\varepsilon'_f\,(2\,N_f)^c + \dfrac{\sigma_f^{'} - k_m\,\sigma_m}{E}\,(2\,N_f)^b - \varepsilon_{at} = 0$
Topper SWT Balda 3	$\dfrac{\sigma_f^{'2}}{E}\,(2\,N_f)^{2\,b} + \sigma_f^{'}\,\varepsilon'_f\,(2\,N_f)^{b+c} - \varepsilon_{at}\,(\sigma_a + k_m\,\sigma_m) = 0$

The methods in the groups vary in the new coefficient k_m. Its values are in tab. 2. It has ascertained that the method Balda 3 is a special form of the Bergmanns' method [3].

Table 2. Values of the coefficient k_m

Authors	k_m
Crews & Hardrath, Morrow, Topper	0
Landgraf, Morrow-Landgraf, SWT	1
Balda 1, 2, 3	0.5

4. New approaches to LCF testing and data processing

Attempts to shorten a period needed for getting LCF parameters are occurring rather often in recent time. The reasons are obvious – to accelerate a development of new products and to make tests cheaper. The main idea for finding the unknown material parameters from any LCF test is based on the following assumptions:

- The relative damage $d_k = d(\sigma_{ak}, \boldsymbol{p}) = 1/N_{fk}$ caused by the kth stress cycle depends on a stress intensity expressed by a stress amplitude σ_{ak} and a vector of material parameters $\boldsymbol{p} = [\,\sigma_f^{'}, b, \varepsilon'_f, c\,]$, where N_{fk} is a number of cycles to failure under σ_{ak}.

- The relative total damage $d_t = 1$, when a failure occurs.
- The law of linear damage cumulation holds, i.e. $d_t = \sum_{\forall k} d_k$.

A term *damage* occurs rather often, when dealing with fatigue life calculations. If a concept of linear cumulation of damage were accepted, the damage caused by the kth stress/strain cycle would be $d_k = 1/N_{fk}$ irrespective of by which method the number $N_{fk} = N_f(\sigma_{ak}, \varepsilon_{apk})$ were obtained. The last assumption enables to use the Pålmgren-Miner criterion for a damage estimation by minimizing the difference $r_\nu = [1 - \sum_{\forall k} d_k]_\nu$ for each νth specimen. Should data from $\nu = 1, \ldots, n$ specimens be processed simultaneously, n residuals r_ν, one for each specimen, were generated. They create a set of n nonlinear algebraic equations, a solution of which is definite in case that $n \geq 4$. If $n < 4$, the system is under-determined with many solutions. In any case, it is possible to search a solution $\boldsymbol{p}^*$, which is closest to the initial estimate $\boldsymbol{p}_0$ by minimizing a sum of squares of residuals

$$S(\boldsymbol{p}) = \sum_{\nu=1}^{n} \left[1 - \sum_{\forall k} d_{\nu k}(\sigma_{ak}, \sigma_{mk}, \boldsymbol{p}) \right]^2 , \tag{16}$$

that tends to zero at the point $\boldsymbol{p}^*$. This approach may be applied to any LCF test. For the purpose, the computer program LCFide has been built in the MATLAB language. Its standard procedure fminsearch used for the mimimization of the function (16) is based on the Nelder-Mead simplex method [9]. The function is robust and does not need derivatives of the function $S(\boldsymbol{p})$.

4.1. Processing of LCF vibration data

The first accelerated tests were performed on a steel bar clamped and excited to resonance vibrations in the middle of its length [7]. The frequency of vibrations was fixed and closed to 1 kHz. The amplitudes of vibration were strongly non-stationary due to detuning the bar natural frequency caused by a proceeding damage during the test. While the exciting frequency was constant and equal the initial resonant frequency, intensity of vibrations was dropping in spite of an attempt to control manually the exciter power to keep stress amplitudes constant. The stress intensity was measured by a strain gauge and recalculated to a local stress in the critical cross section of the bar. Amplitudes σ_a of the stress were measured with a period of 2 seconds until the bar broke. This caused a splitting of the stress time history into N_b blocks with an equal number of cycles and unequal stress amplitudes in each. The only thing has been known, besides the time series of stress amplitudes σ_a, namely that relative damage $d_t = 1$ at the end of the test. Those were the initial conditions for data processing.

The printer output of the LCFide program run, and the subsequent Fig. 1 show the results of the processing of one vibration test of a slender bar. The first subfigure shows the measured time series of local stress amplitudes (highest), stress amplitudes measured by strain gauge (dotted), accelerometer data, calculated quotient of accelerometer and strain-gauge data (lowest). The second subfigure shows the calculated relative damages generated in blocks (line with peaks), and the cumulated relative damage for the parameters estimated due to [5]. The last subfigure contains the same for the optimized parameters.

```
@@@@@@@@ LCFide 25-Aug-2009 @@@@@@@@

          material   =     steel =>
          E   [MPa] = 210000.0 =>
          Rm  [MPa] =     569.0 =>

     sigma_f' [sfa] =    853.5 [MPa]
     b        [b  ] = -0.0870
     eps_f'   [efa] =   0.5900
     c        [c  ] = -0.5800
     K'       [Ka ] =    923.8 [MPa]
     n'       [na ] =   0.1500

     sigma_c  [sc ] =    256.1 [MPa]
     eps_c    [ec ] =   0.0014
     w        [Nc ] = 11.4943
     N_c      [Nc ] =   511765

          iprint     =       50 =>
          frekvence =   915.00 =>
          převod  p =    4.319 =>
  ------------------------------

  Original parameters
  Elapsed time      =     1.78 [s]

                              damage

  Crews-Hardrath:              2.933
  Landgraf:                    2.933
  Morrow:                      2.933
  Morrow-Landgraf:             2.933
  Topper:                      2.933
  Smith-Wetzel-Topper:         2.933
  Balda 1:                     2.933
  Balda 2:                     2.933
  Balda 3:                     2.933

************************************
  itr  nfJ   sum(r^2)        x
************************************

   0    0  3.7371e+000  1.0000e+000
                        1.0000e+000
                        1.0000e+000
                        1.0000e+000
                        1.0000e+000

  50   50  3.3174e-004  1.0090e+000
                        8.9617e-001
                        1.0586e+000
                        1.0276e+000
                        1.0445e+000

 100  100  2.4579e-007  1.0072e+000
                        8.9624e-001
                        1.0609e+000
                        1.0275e+000
                        1.0459e+000

 110  110  6.5625e-008  1.0072e+000
                        8.9629e-001
                        1.0608e+000
                        1.0275e+000
                        1.0458e+000

************************************
    b         c       sfa      efa       na      Ka        E

-0.08700 -0.58000  853.50  0.59000  0.15000  923.80  210000  B&S estimates
-0.08762 -0.51985  905.43  0.60621  0.16856  985.13  219624  optimized pars

Elapsed time is 19.213006 seconds.
  ------------------------------

  Optimized parameters
  Elapsed time      =     1.61 [s]

                              damage

  Crews-Hardrath:              1.000
  Landgraf:                    1.000
  Morrow:                      1.000
  Morrow-Landgraf:             1.000
  Topper:                      1.000
  Smith-Wetzel-Topper:         1.000
  Balda 1:                     1.000
  Balda 2:                     1.000
  Balda 3:                     1.000

@@@@@@@@@@ tycka112-08.txt @@@@@@@@@@@
```

The program LCFide enables the user to choose the way how to process the supplied data interactively. Vibration data may come in the format for MS Excel or as text files. Contingent missing data are linearly interpolated.

Estimates of material parameters are either input from a file, or evaluated from the Bäumler-Seeger formulae [5] on the basis of Young's modulus and a strength R_m of the tested material. As seen from the output sheet, the second possibility was chosen. It is apparent that the fatigue life evaluated from the parameters obtained in such a way is about 3 times underestimated, because relative damages calculated are equal 2.933. The reason why all 9 methods delivered identical results is zero mean stress during vibrations.

Intermediate results of the optimization process applied for the vector $\bar{\boldsymbol{p}} = [\bar{p}_i]$, $\bar{p}_i = p_i/p_{i0}$ of parameters $\boldsymbol{p} = [p_i] = [\,b, c, \sigma_f', \varepsilon_f', E\,]$ are displayed with a chosen step of 50 iterations. The Young's modulus E has been also put among identified parameters, because its value has not been measured in advance. The required tolerance of the solution has been reached after 110 iterations.

The displayed times spent for various steps of the program run belong to the PC with Intel Duo Q6600, 2.4 GHz.

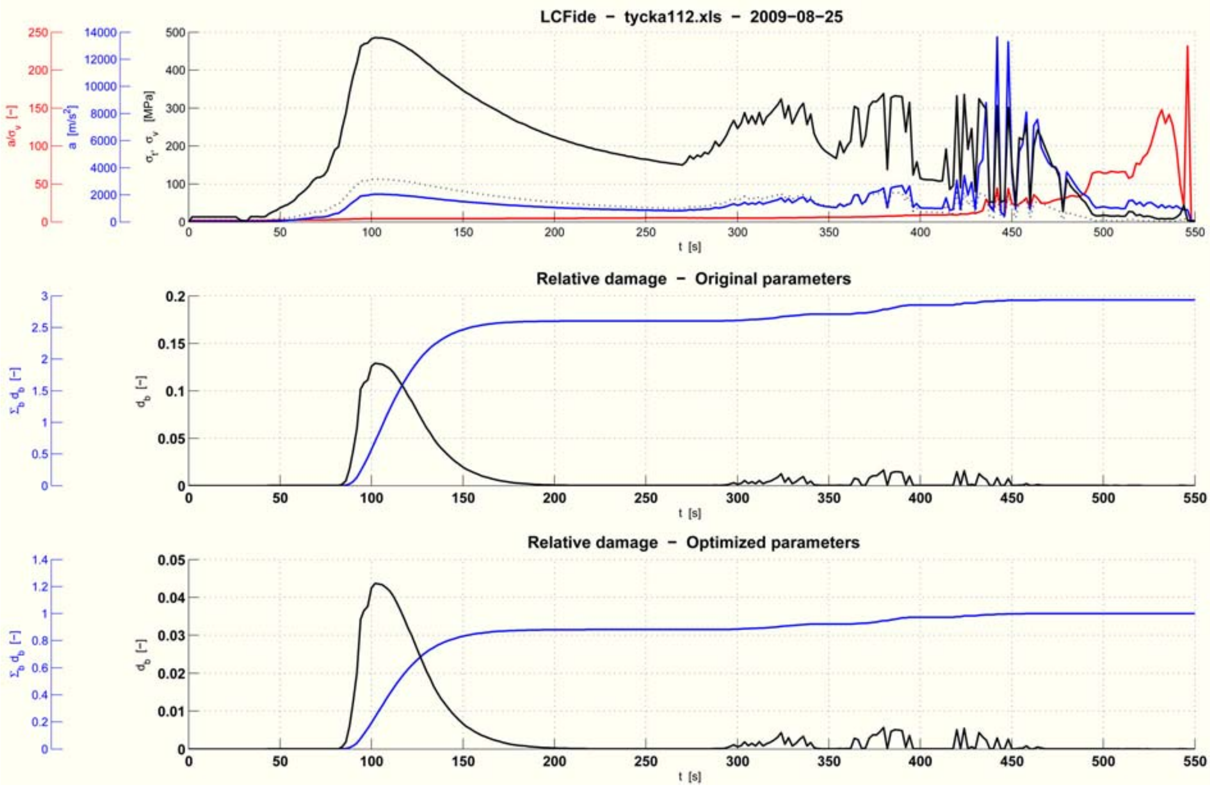

Fig. 1. Vibrational data (a), estimated damage (b), optimized damage (c)

4.2. Processing of nonstationary LCF tests

Any LCF test may be processed in a similar way as the vibration test. It is important that stress amplitudes σ_a should go through the whole interval of damaging stresses.

A gradual test runs with stress varying step by step with constant amplitudes σ_a and known numbers N_f in blocks. Random tests should fulfill other conditions. A random process $\sigma_a(t)$ should be sampled by a sampling frequency $f_s \approx 20 f_h$, where f_h is the highest frequency in the stress process. In this case, the relative error in the peak measurement will be about 1% in the highest frequency component and lower for others.

4.3. New processing of classical LCF tests

The classical LCF test data contain all information needed for the identification of the LCF material parameters by the optimization procedure. The advantage of this approach would be that all available data be processed as mutually joint, while the classical way of processing described by equations (7) solves the parameters for stress and strain separately.

The unifying element between stress and strain is damage, which needs all LCF material parameters for its evaluation regardless of stress or strain is used. When using both, the number of measurements and corresponding residuals doubles due to a double output from the tests (stress and strain). Of course, that this could contribute to a better parameter estimation.

A practical application of the method have shown that processed data should have low scattering and an initial guess of sought parameters $\boldsymbol{p}$ be rather good. If the conditions are not fulfilled, the method may collapse due to numerical instability.

5. Conclusion

The paper describes new ideas in the low cycle fatigue testing and data processing. The aim of the research has been focused on accelerating and making the tests cheaper without spoiling the quality of results. A new method of LCF data processing based on the evaluation of a relative damage has been proposed and tested on a series of different low cycle fatigue tests. Material low cycle fatigue parameters are searched by means of the optimization procedure applied for the minimization of a sum of squared differences of calculated and measured relative damages. Practical applications of the method have revealed that fatigue lives calculated from the estimated LCF coefficients may differ by orders from those obtained from the identified parameters.

Acknowledgements

The work has been supported by the Research Plan of the Ministry of Education, Youth and Sports of the Czech Republic MSM4771868401. Reviewers' comments and recommendations, which contributed to the clarity of the article, are also acknowledged.

References

[1] ASTM, Manual on Low Cycle Fatigue Testing, ed. R. M. Wetzel. ASTM Special Technical Publication 465, Baltimore, 1969, ISBN 8031-0023-X.

[2] Balda, M., Identification of low cycle fatigue parameters (In Czech), Res. Rpt. VYZ 1178/09, ŠKODA RESEARCH Ltd, Plzeň, 2009.

[3] Bergmann, J. W., Zur Betriebsfestigkeitsbemessung gekerbter Bauteile auf der Grundlage der örtlichen Beanspruchungen, Institut für Stahlbau und Werkstoffmechanik, TH Darmstadt, 1983.

[4] Basquin, O. H., The Exponential Law of Endurance Tests. *Proceedings*, American Society for Testing and Materials, ASTEA, Vol. 10, 1910, pp. 625–630.

[5] Bäumel, A., Seeger, T., Materials Data for Cyclic Loading – Supplement 1. Materials Science Monographs 61, Elsevier Science Publishers, Amsterdam, 1990.

[6] Coffin Jr., L. F., A Study of the Effects of Cyclic Thermal Stresses on a Ductile Metal. Trans. ASME, Vol. 76, 1954, pp. 931–950.

[7] Hyrát, J., Horák, V., Damaging of Parts under Higher Frequencies (In Czech), Res. Rpt. VYZ 0933/06, ŠKODA Research Ltd, Plzeň, 2006.

[8] Manson, S. S., Behavior of Materials under Conditions of Thermal Stress, Heat transfer Symp., University of Michigan, Engineering Research Inst., 1953.

[9] Nelder, J. A., Mead, R., A simplex method for function minimization, Computer Journal, 1965, Vol. 7, pp. 308–313.