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A computational investigation of vibration attenuation of a rigid rotor turning at a variable speed by means of short magnetorheological dampers

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Abstract

Rotors of all rotating machines are always slightly imbalanced. When they rotate, the imbalance induces their lateral vibration and forces that are transmitted via the bearings into the foundations. These phenomena are significant if the rotor accelerates or decelerates and especially if it passes over the critical speeds. The vibration can be reduced if the rotor supports are equipped with damping elements. To achieve optimum performance of the damper, the damping effect must be controllable. At present time, semiactive magnetorheological squeeze film dampers are a subject of intensive research. They work on a principle of squeezing a thin film of magnetorheological liquid. If magnetic field is applied, the magnetorheological liquid starts to flow only if the shear stress between two neighbourhood layers exceeds a limit value which depends on intensity of the magnetic field. Its change enables to control the damping force. In the mathematical models, the magnetorheological liquid is usually considered as Bingham one. Application of the computer modelling method for analysis of rotors supported by rolling element bearings and magnetorheological squeeze film dampers and turning at variable angular speed requires to set up the equations of motion of the rotor and to develop a procedure for calculation of the damping force. Derivation of the equations of motion starts from the first and second impulse theorems. The pressure distribution in the thin lubricating film can be described by a Reynolds equation modified for the case of Bingham liquid. In cavitated areas, it is assumed that pressure of the medium remains constant. The hydraulic force acting on the rotor journal is then obtained by integration of the pressure distribution around the circumference and along the length of the damper. Applicability of the developed procedures was tested by means of computer simulations and influence of the control of the damping force on vibration of the rotor was analyzed.

Keywords: accelerating rotors, magnetorheological dampers, critical revolutions, vibration damping

1. Introduction

Because of manufacturing and assembling inaccuracies, rotors of all rotating machines are always slightly imbalanced. When they rotate, the imbalance produces their lateral vibration and the forces that are transmitted via the bearings into the stationary part and into the foundations. This results into reduction of the service life of all machine components, into the noise increase and in propagation of vibration and mechanical waves into the ambient space. These effects are significant especially if the rotors pass the critical speeds during their starting or running out.

The vibration amplitude and magnitude of the forces transmitted via the bearings are considerably influenced by a flexible suspension and by damping elements placed between the rotor and the stationary part. The damping effect depends on modal properties of the rotating machine and on the operation conditions. To achieve the optimum compromise between amplitude

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of the vibration and the forces transmitted through the bearings, it is desirable to use dampers that enable to control magnitude of the damping effect and thus to adapt their performance to the current operating conditions.

At present time, a considerable attention is paid to investigation of controllable semiactive magnetorheological squeeze film dampers because their application has a number of advantages. Their design is simple, they do not require a complicated and expensive feedback control system, they work on the principle of dissipation of mechanical energy and therefore they do not destabilize vibration of the rotor. In addition, their mounting and maintenance is not much demanding.

Even if the problem of passing the rotors over the critical revolutions is very important from the technological point of view, almost all publications and computer procedures from the field of rotor dynamics are related only to the case when the rotor turns at a constant angular speed. In [1, 2, 3, 4], there are given the equations of motion of a flexibly supported disc performing a spherical movement. The system has only two degrees of freedom and its vibration is described by two equations. The equations of motion of a symmetric rotating disc having four degrees of freedom have been derived in [5]. Using assumption of small displacements and rotations, the authors simplified the Euler dynamic equations. But the resulting relationships are referred again only to the disc rotating at constant angular speed. In addition, unfortunately, some authors analyzing the rotors turning at variable speed use incomplete or even wrong equations of motion [6, 7].

At present time, the magnetorheological dampers are a subject of intensive experimental and theoretical research in the field of rotor dynamics. In [8], Wang and Meng studied experimentally the vibration properties and the control method of a flexible rotor supported by a magnetorheological squeeze film damper. In [9], Forte et al. presented results of the theoretical and experimental investigation of a long magnetorheological damper. In [10], Wang et al. developed a mathematical model of a long squeeze film magnetorheological damper based on modification of a Reynolds equation in which the authors included influence of the change of the width of the gap around the damper circumference on the yield shear stress of the magnetorheological liquid.

The principal intention of this article is to derive the equation of motion of a statically unbalanced rigid rotor turning at variable angular speed which is supported by rolling-element bearings and short magnetorheological squeeze film dampers and to test the influence of control of the yielding shear stress of the magnetorheological lubricant on the damping effect during passing the rotor over the critical revolutions.

2. Determination of the damping force of a short magnetorheological squeeze film damper

The magnetorheological squeeze film dampers (fig. 1) are semiactive damping devices. The damping effect is reached by squeezing a thin film of magnetorheological liquid whose resistance against the flow is controlled by intensity of the magnetic field. Magnetorheological fluids are colloidal suspensions consisting of carrying liquid (usually oil), tiny ferromagnetic particles dispersed in it and of a stabilizing agent that prevents coupling the particles due to action of the van der Waals surface forces. If the magnetorheological liquid is not effected by magnetic field, the ferromagnetic particles are dispersed in a random way and the liquid behaves as newtonian one. But when the magnetic field is applied, the particles start to form small chains and the flow occurs only if the shear stress between two neighbouring layers exceeds a limit value

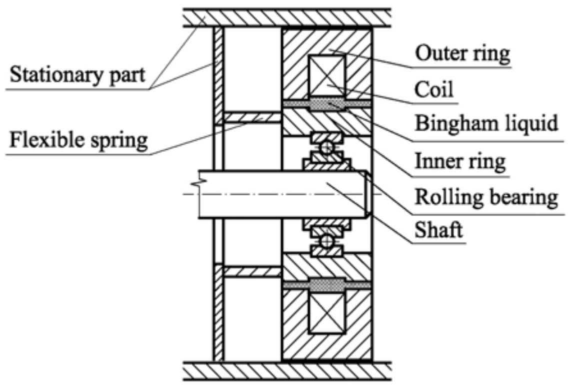

Fig. 1. Scheme of a MR damper

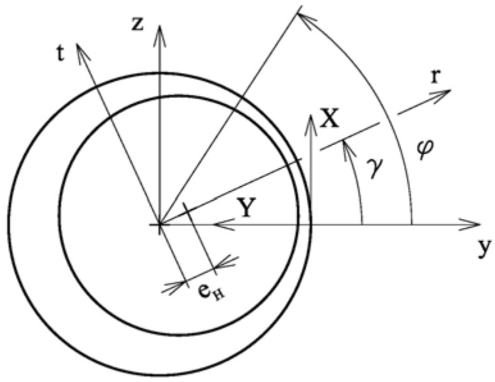

Fig. 2. The damper coordinate system

(yield shear stress). Its magnitude depends on intensity of the magnetic flux and this enables to control the damping effect.

It is assumed in the mathematical model, that (i) the inner and outer rings of the damper are absolutely rigid and smooth, (ii) the width of the damper gap is very small relative to the radii of both rings, (iii) ratio of the length of the damper to the diameter of its outer and inner rings is small, therefore it can be considered as short, (iv) the lubricant is Bingham liquid, (v) the yield shear stress depends on magnitude of the magnetic induction, (vi) the flow in the oil film is laminar and isothermal, (vii) pressure of the lubricant in the radial direction is constant, (viii) the lubricant is considered to be massless and (ix) influence of the curvature of the oil film is negligible.

Distribution of the pressure gradient in the lubricating film is described by a Reynolds equation modified for the case of Bingham liquid. Its derivation for a short damper has been performed in [11]

$$h^3 p'^3 + 3\left(h^2 \tau_y - 4\eta \dot{h} Z\right) p'^2 - 4\tau_y^3 = 0 \quad \text{for} \quad \dot{h} > 0, \tag{1}$$

$$h^3 p'^3 - 3\left(h^2 \tau_y + 4\eta \dot{h} Z\right) p'^2 + 4\tau_y^3 = 0 \quad \text{for} \quad \dot{h} < 0. \tag{2}$$

Z is the axial coordinate, p' is the pressure gradient in the axial direction, τ_y, η are the yield shear stress and viscosity of the Bingham liquid and h is thickness of the lubricating film. Dot $(\dot{\ })$ denotes the first derivative with respect to time.

Determination of the pressure gradient as a function of the axial coordinate Z leads to solving cubic algebraic equations (1) or (2). Solution of each of them gives three roots. The searched one must be real (not complex).

The yield shear stress τ_y is proportional to the square of the magnetic induction B [8, 12]

$$\tau_y = k_\tau B^2. \tag{3}$$

Parameter k_τ is a material constant which depends on concentration of the ferromagnetic particles and on other liquid properties. Its magnitude is either given by the manufacturer or it must be determined experimentally for the concrete magnetorheological fluid.

For the simplest design of the dampers, it holds with sufficient accuracy

$$B = \mu^2 N_C \left(\frac{I}{2h}\right)^2. \tag{4}$$

I is the electric current, μ is permeability of the magnetorheological liquid and N_C is number of the coil turns. In more complicated cases the current – magnetic induction relationship must be determined e.g. by application of a finite element method.

Thickness of the lubricating film depends on position of the inner damper ring relative to the outer one

$$h = c - e_H \cos(\varphi - \gamma). \tag{5}$$

c is the width of the gap between the inner and outer rings of the damper, e_H is the journal eccentricity, φ is the circumferential coordinate and γ is the position angle of the line of centres (fig. 2).

The pressure is obtained by integration of the pressure gradient

$$p = \int p' \,\mathrm{d}Z \tag{6}$$

with the boundary condition

$$p = p_A \quad \text{for} \quad Z = \pm\frac{L}{2}. \tag{7}$$

p_A is the pressure in the surrounding space (atmospheric pressure) and L is the length of the damper.

If the pressure drops to a critical value, a cavitation takes place. It is assumed that pressure of the medium in cavitated areas remains constant. Then it holds with sufficient accuracy

$$p_d = p \qquad \text{for} \quad p \geq p_{CAV}, \tag{8}$$

$$p_d = p_{CAV} \quad \text{for} \quad p < p_{CAV}. \tag{9}$$

p_d is the pressure distribution in the layer of lubricant and p_{CAV} is the pressure in cavitated regions.

Components of the damping force are obtained by integration of the pressure distribution around the circumference and along the length of the damper

$$F_{dy} = -R \int_0^{2\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} p_d \cos\varphi \,\mathrm{d}Z \,\mathrm{d}\varphi, \qquad F_{dz} = -R \int_0^{2\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} p_d \sin\varphi \,\mathrm{d}Z \,\mathrm{d}\varphi. \tag{10}$$

F_{dy}, F_{dz} are y and z components of the damping force, R is the inner ring radius.

3. Derivation of the equations of motion of a rigid statically unbalanced rotor

It is assumed that the investigated rotor is absolutely rigid and is supported by rolling element bearings mounted with cage springs which are coupled with the casing. Damping of the system is realized by magnetorheological squeeze film dampers placed between the cage springs and the stationary part. The rotor rotates at variable angular speed and is loaded by its weight. In addition, it is excited by its imbalance. The stationary displacement of the rotor caused by its weight is eliminated by prestressing the cage springs. The squeeze film dampers are implemented into the mathematical model by means of nonlinear force couplings. As their length to diameter ratio is assumed to be small, they are considered as short in the analysis.

To describe position of the rotor, three coordinate systems are used (fig. 3). The first one $(Oxyz)$ is fixed. Its axis x is coincident with the centre line of the rotor in undeformed state and connects centers of the bearings. Axes x', y', z' of the second coordinate system are parallel

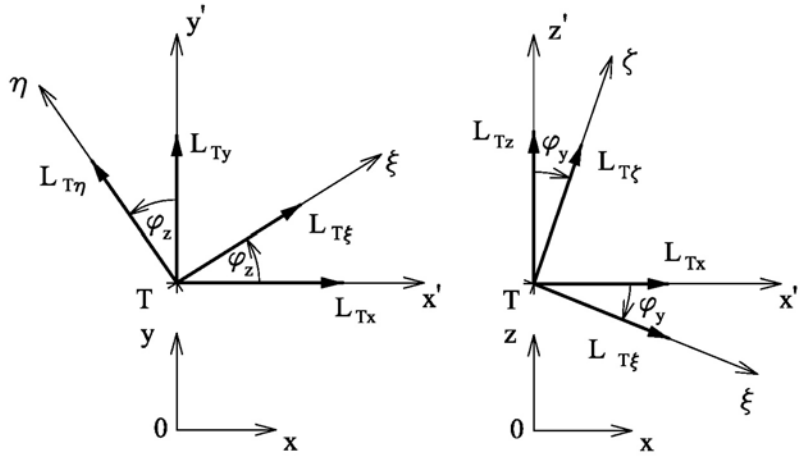

Fig. 3. Coordinate systems and components of the rotor moment of momentum L_{Tx}, L_{Ty}, L_{Tz}, $L_{T\xi}$, $L_{T\eta}$, $L_{T\zeta}$ denote components of the moment of momentum of the rotor

with corresponding axes x, y and z and its origin lies in the centre of gravity T of the rotor which is because of imbalance slightly shifted from the rotor axis. The third coordinate system has its origin in the centre of gravity T and its axes are denoted ξ, η, ζ. It slides together with the frame of reference $Tx'y'z'$ and slightly rotates about axes y' and z' so that its plane $\eta\zeta$ remains perpendicular to the axis of the rotor.

Making use of fig. 3, the equations of motion of the rotor are derived from the first and second impulse theorems

$$\frac{\mathrm{d}}{\mathrm{d}t}(m\dot{y}_T) = F_{Cy}, \qquad \frac{\mathrm{d}}{\mathrm{d}t}(m\dot{z}_T) = F_{Cz}, \tag{11}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(J_D\dot{\varphi}_y + \omega J_A\varphi_z) = M_{Cy}, \qquad \frac{\mathrm{d}}{\mathrm{d}t}(J_D\dot{\varphi}_z - \omega J_A\varphi_y) = M_{Cz}. \tag{12}$$

m, J_D, J_A are the mass and moments of inertia (diametral and axial) of the rotor, y_T, z_T are the horizontal (y) and vertical (z) displacements of the rotor centre of gravity, φ_y, φ_z are small rotations of the rotor about axes y, z, ω is angular speed of the rotor rotation, F_{Cy}, F_{Cz} are the sum of components of all forces acting on the rotor (applied, damping, elastic produced by deformation of the cage springs, rotor weight) in the horizontal and vertical directions, M_{Cy}, M_{Cz} are the sum of components of all moments acting on the rotor relative to its centre of gravity (which is because of small deformations the same as to the rotor centre) in the y and z directions, t is time.

After performing the differentiations and taking into account the relations between displacements of the rotor centre (displaced point O) and its centre of gravity T

$$y_T = y + e_T\cos\varphi_T, \tag{13}$$
$$z_T = z + e_T\sin\varphi_T, \tag{14}$$

the equations of motion obtain the form

$$m\ddot{y} = F_{Cy} + m\cdot e_T\varepsilon\sin\varphi_T + m\cdot e_T\omega^2\cos\varphi_T, \tag{15}$$
$$m\ddot{z} = F_{Cz} - m\cdot e_T\varepsilon\cos\varphi_T + m\cdot e_T\omega^2\sin\varphi_T, \tag{16}$$

$$J_D\ddot{\varphi}_y + \omega J_A\dot{\varphi}_z + \varepsilon J_A\varphi_z = M_{Cy}, \quad (17)$$

$$J_D\ddot{\varphi}_z - \omega J_A\dot{\varphi}_y - \varepsilon J_A\varphi_y = M_{Cz}. \quad (18)$$

y, z are the horizontal and vertical displacements of the rotor centre, e_T, φ_T are eccentricity and position angle of the rotor centre of gravity and ε is angular acceleration of the rotor rotation. Dots ($\ddot{\ }$) denote the second derivative with respect to time.

4. Methodology of determination of the current – magnetic induction relationship

The magnetorheological damper presented in [9] was used to demonstrate the methodology of determination of the current – magnetic induction relationship.

The schema of the investigated damper is drawn in fig. 4. The damper consists of one outer and one inner ring, two electric coils and their casings, a flexible spring and of a ball bearing which is mounted with a sleeve made of aluminium alloy and coupled with the shaft. The gap between the rings is filled with magnetorheological liquid whose properties are controlled by intensity of the magnetic field produced by the coils supplied with electric current. The rubber seals are used to prevent leakage of the liquid from the damper gap. The ends of the coil windings are connected in such way that the current in each of them flows in the opposite direction.

The body of the damper, its rings, the bearing and the shaft are made of steel having relative permeability 2 000. Further it is assumed that relative permeability of the magnetorheological liquid is 5 and of the air and of all other mechanical parts of the damper is equal to 1. Each coil has 240 turns of a copper conductor of 0.63 mm in diameter. This enables the maximum current of 2 A.

The magnetostatic analysis of the damper was performed by means of a finite element code COMSOL Multiphysics. Because the length to diameter ratio of the damper is small, the flow in the thin film of the magnetorheological liquid prevails in the axial direction. This enables to consider it in the radial cross section of the damper gap as planar and therefore also the distribution of the magnetic induction in the film can be investigated by means of a 2D magnetostatic analysis. Dependence of the magnetic induction on intensity of the magnetic field is assumed

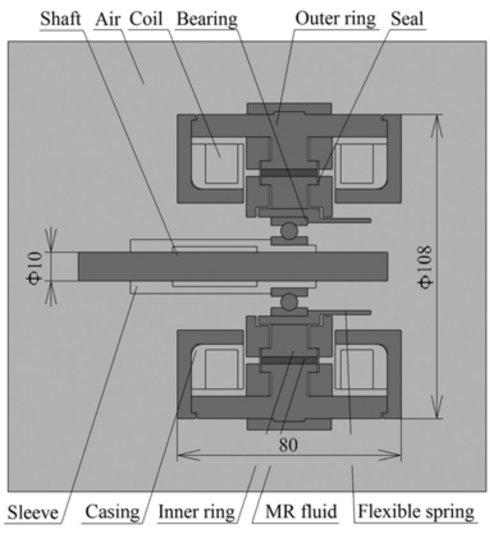

Fig. 4. Scheme of the investigated MR damper

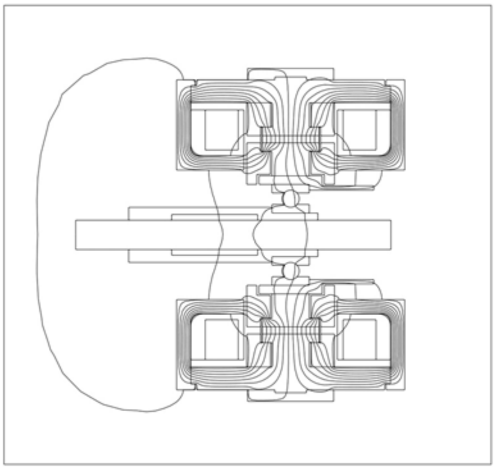

Fig. 5. The magnetic field distribution, (2.5–2.5) mm, 1 A

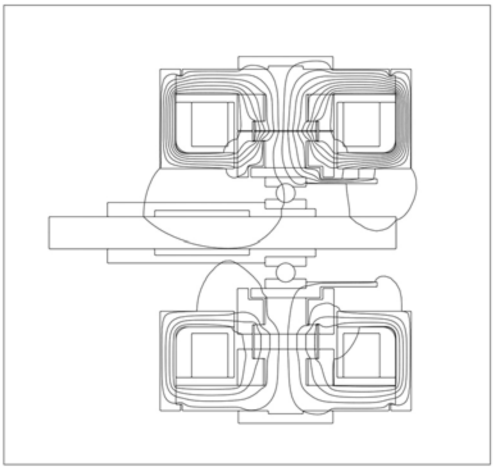

Fig. 6. The magnetic field distribution, (0.25–4.75) mm, 2 A

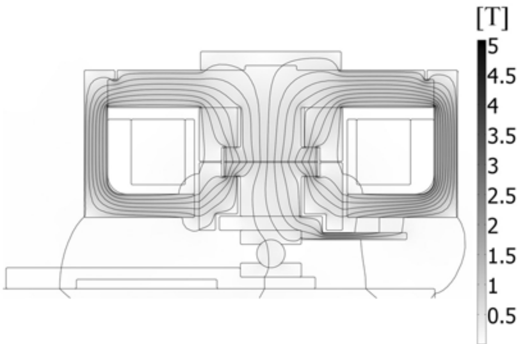

Fig. 7. The magnetic field, 0.25 mm, 2 A

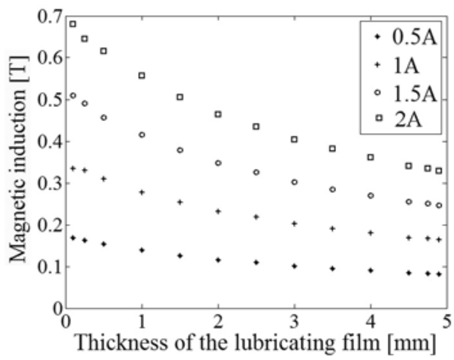

Fig. 8. The flux density – thickness relationship

to be linear and isotropic for all materials. Magnetic saturation and material hysteresis are neglected. The source of the electric current is defined by a current density at locations of the electric coils ($1.76 \cdot 10^6$ Am^{-2} for the current of 1 A). In sufficient distance from the damper, the boundary condition of the magnetic insulation is defined.

The area was discretized in approximately 20 000 triangular elements with Lagrange-quadratic shape functions. Calculation of the magnetic quantities in the individual nodes arrived at solving a set of linear algebraic equations.

Fig. 5 and fig. 6 show a distribution of the magnetic flux lines for concentric and eccentric positions of the inner damper ring relative to the outer one. Inside the lubricating film the magnetic field is almost homogeneous (fig. 7) and magnitude of its induction is strongly effected by the thickness of the lubricating film.

The dependence of the mean value of the magnetic induction calculated by means of a finite element method in the damper gap (slight variations along the length of the damper have been averaged) on the thickness of the layer of the magnetorheological liquid for several magnitudes of the electric current in the coils is drawn in fig. 8. As evident the magnetic induction is with sufficient accuracy proportional to the current in the coils. Its dependence on the thickness of the lubricating film was approximated by relationship (19)

$$B = \frac{1}{X_1 + X_2 h} I \tag{19}$$

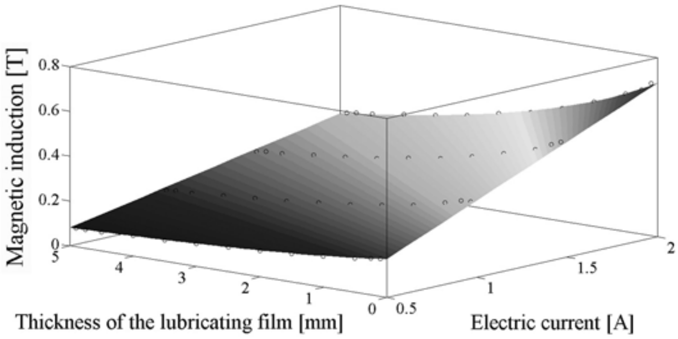

Fig. 9. Approximation of the flux density

where X_1 and X_2 are constants. Employing a least square method, one obtains the values of both constants $X_1 = 0.343\,1$ AT^{-1} and $X_2 = 0.668\,1$ AT^{-1}m^{-1}. Fig. 9 shows that application of relationship (19) approximates dependence of the magnetic induction in the damper gap on the current in the coils with good accuracy.

5. Example

The investigated rotor (fig. 10) is supported by two rolling element bearings mounted with cage springs which are coupled with the casing. The damping in the supports is realized by magnetorheological squeeze film dampers placed between the cage springs and the housing. The rotor rotates at a constant angular speed of 50 rad/s. At a specified moment of time it starts to accelerate (a sine ramp) and when the desired speed of 400 rad/s is reached it continues to turn at constant revolutions (fig. 11). The rotor is loaded by its weight and in addition it is excited by its imbalance. To eliminate deformation caused by the rotor weight, the cage springs are prestressed. The task was to investigate the influence of the dampers on the rotor vibration and on the force transmitted into the foundations.

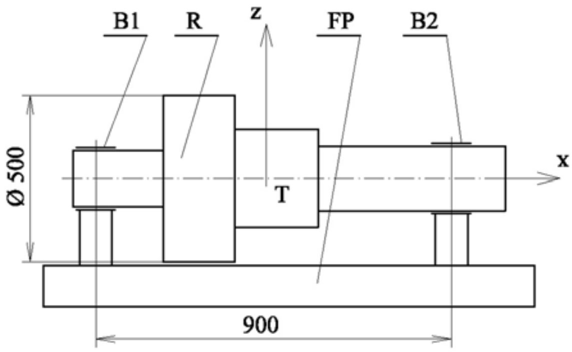

Fig. 10. Scheme of the rotor

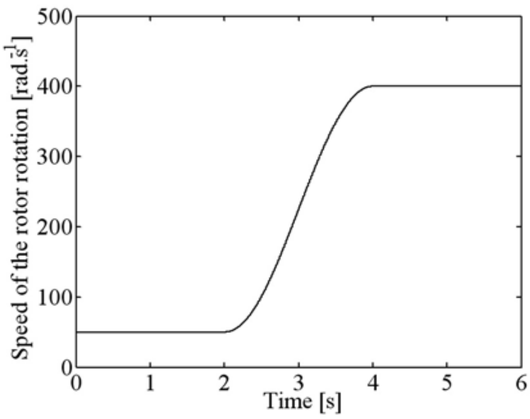

Fig. 11. Time history of the rotor revolutions

The Campbell diagram (fig. 12) shows that the rotor should pass the critical speeds during the acceleration once, maybe twice.

Time history of horizontal vibration of the rotor centre and of the damping force transmitted via damper B2 for two magnitudes of the electric current in the dampers coils are drawn in fig. 13, 14, 15 and 16. It is evident that the higher current produces magnetic field of higher

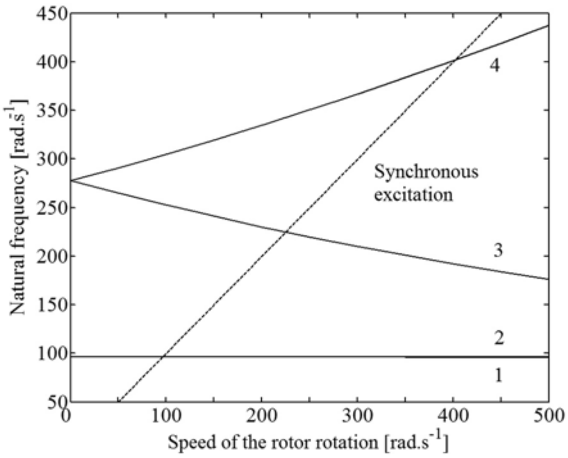

Fig. 12. Campbell diagram

intensity and this results in larger reduction of the rotor vibration in the neighbourhood of the critical revolutions. On the other hand it increases magnitude of the force transmitted into the foundations. The counter-rotating critical speed has no influence on the rotor vibration.

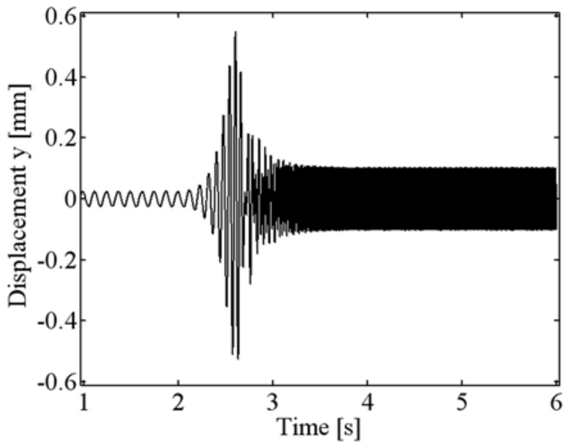

Fig. 13. Horizontal displacement of the rotor (1 A)

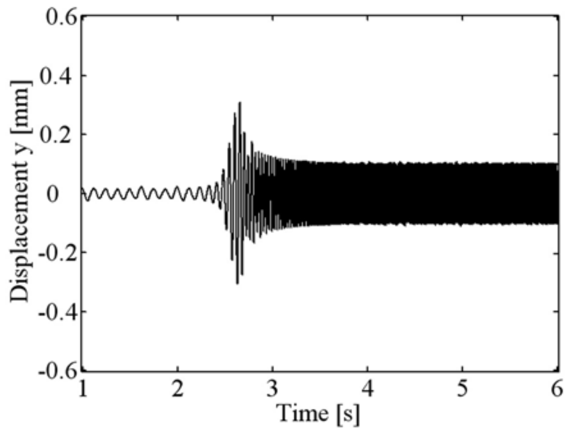

Fig. 14. Horizontal displacement of the rotor (2 A)

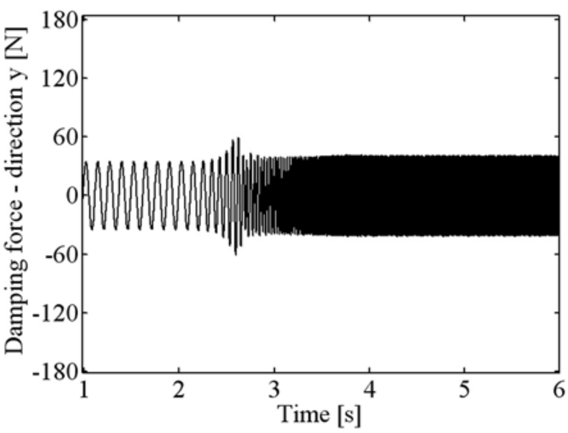

Fig. 15. Force transmitted to the foundations (1 A)

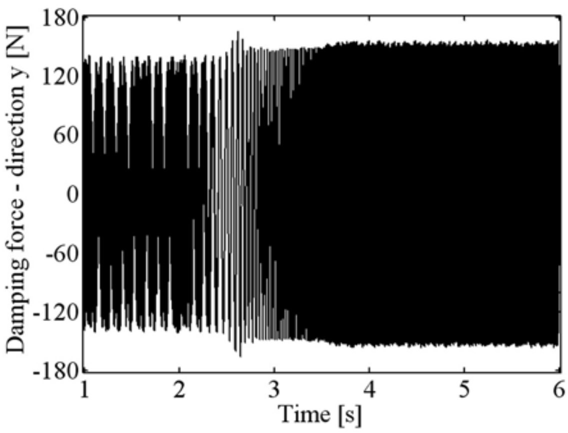

Fig. 16. Force transmitted to the foundations (2 A)

To achieve optimum performance of the damping device, the damping effect can be controlled by the current in the coils. The current is risen only during passing the critical speed and then it is decreased again (fig. 17). Maximum amplitude of the vibration is reduced (fig. 18) and increase of the damping force occurs only for a short time interval when the rotor overcomes the critical revolutions.

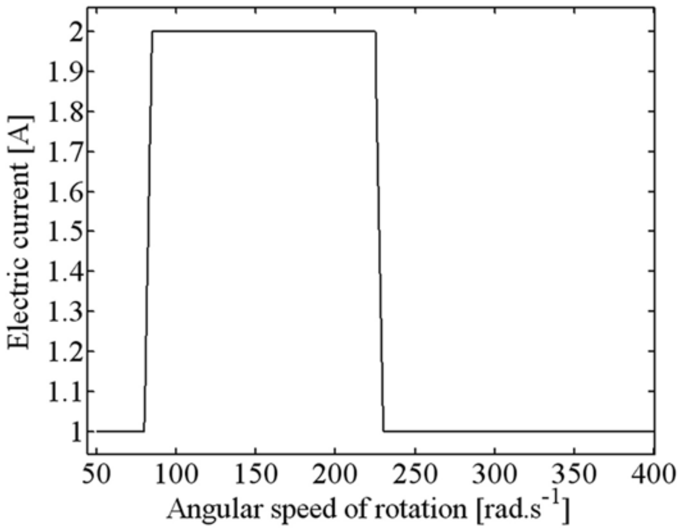

Fig. 17. The current – speed of rotation relationship

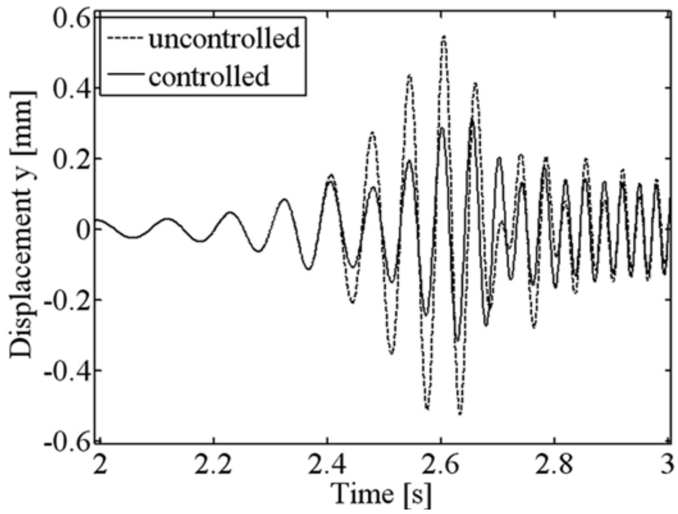

Fig. 18. The rotor horizontal displacement (controlled)

6. Conclusion

The developed numerical procedure represents a computational method for investigation of lateral vibration of rigid rotors that are supported by nonlinear short magnetorheological squeeze film dampers and that rotate at variable angular speed. The change of the yield shear stress of magnetorheological liquid by application of the magnetic field enables to control magnitude of the damping force and thus to achieve optimum performance of the damping device. The procedure is intended for determination of the rotor system response on force and kinematic excitation of a general time history and especially for investigation of the rotor vibration during its passing over the critical revolutions. The results of the carried out computer simulations showed that the damping effected both amplitude of the vibration and the force transmitted into the stationary part and that it was most efficient in the neighbourhood of the critical speeds. The dependence of magnitude of the damping force on magnitude of the electric current which produces the magnetic field has been also proved.

Acknowledgements

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