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Analytical and numerical investigation of trolleybus vertical dynamics on an artificial test track

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Abstract

Two virtual models of the ŠKODA 21 Tr low-floor trolleybus intended for the investigation of vertical dynamic properties during the simulation of driving on an uneven road surface are presented in the article. In order to solve analytically vertical vibrations, the trolleybus model formed by the system of four rigid bodies with seven degrees of freedom coupled by spring-damper elements is used. The influence of the asymmetry of a sprung mass, a linear viscous damping and a general kinematic excitation of wheels are incorporated in the model. The analytical approach to solving the ŠKODA 21 Tr low-floor trolleybus model vibrations is a suitable complement of the model based on a numerical solution. Vertical vibrations are numerically solved on the trolleybus multibody model created in the *alaska* simulation tool. Both virtual trolleybus models are used for the simulations of driving on the track composed of vertical obstacles. Conclusion concerning the effects of the usage of the linear and the nonlinear spring-damper elements characteristics are also given.

Keywords: vehicle dynamics, trolleybus, analytical solution, numerical simulation, multibody model

1. Introduction

Computational models of vehicles, which are used in vehicle dynamics tasks, can be of a various complexity and therefore it is efficient to have a variety of models with respect to their application. General approaches to the vehicle modelling and their reviews can be found in [1] and [12]. The advantage of simple models (concerning kinematic structure and number of degrees of freedom) is mainly the shorter computational time of particular analyses. They can be used for a sensitivity analysis, optimization [15], parameters identification etc. The problems of interaction [5] are also studied very often with this sort of models. On the other hand more complex multibody models [4, 6, 10] can be used for detailed analyses and for the investigation of the chosen structural elements behaviour. The most of published works are based on numerical simulations with the created vehicle models and the analytical methods, which can bring faster and more accurate analyses, are omitted.

In connection with previous contributions to the investigation of vertical vibration of vehicles under various conditions [2, 3, 7, 8, 9, 10, 11, 13, 14, 16, 17] this article deals with the analytical and numerical solutions of vertical vibration of the empty ŠKODA 21 Tr low-floor trolleybus (see fig. 1) models.

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Fig. 1. The ŠKODA 21 Tr low-floor trolleybus

The usage of the simplified analytical model for the dynamic analysis of road vehicles is justified because of the transparency of a mathematical model, easier implementation and the possibility of a better understanding of the mechanical system behaviour. Analytical solution enables to put the monitored quantities (displacements, velocities, accelerations) in the form of continuous function of time (enabling the analytical performing of derivative and integration) in contradiction to the discrete form of those quantities obtained by means of the numerical solution. Relations for the calculation of the monitored quantities in the whole investigated period of time are obtained during the analytical solving of the equations of motion. The numerical method requires to solve the equations of motion for each integrating step of the investigated period of time. In order to solve numerically the vertical vibration the trolleybus multibody model created in the *alaska* simulation tool is used.

The main differences between the models are in the consideration of linear characteristics of spring-damper elements and in the impossibility of including the bounce of the tire from the road surface in the analytical model.

2. Analytical solution

For the analytical solution the trolleybus model is considered to be a system of four rigid bodies with seven degrees of freedom, coupled by spring and dissipative elements (see fig. 2), taking into account a linear viscous damping and the influence of asymmetry (e.g. mass distribution) and with general kinematic excitation of the individual wheels. The rigid bodies correspond to the sprung mass (trolleybus body) and the unsprung masses – the rear axle (including wheels) and the front half axles (including wheels). Spring and dissipative elements model the tire-road surface contact (two front and four rear wheels), the air springs in the axles suspension (two front and four rear ones) and the hydraulic shock absorbers in the axles suspension (two front and four rear ones). As it was already mentioned, characteristics of the spring and dissipative elements are supposed to be linear.

In general case, for the considered trolleybus model it is possible to put the equations of motion in the matrix form (see [14])

$$\mathbf{M} \cdot \ddot{\mathbf{q}}(t) + \mathbf{B} \cdot \dot{\mathbf{q}}(t) + \mathbf{K} \cdot \mathbf{q}(t) = \mathbf{f}(t), \tag{1}$$

where $\mathbf{q}(t) = [\varphi, \theta, z, z_1, z_2, z_3, \varphi_3]^T$, $\dot{\mathbf{q}}(t)$, $\ddot{\mathbf{q}}(t)$ are the vectors of the generalized coordinates (φ is the angular displacement of the trolleybus body – sprung mass – around the longitudinal x-axis, θ is the angular displacement of the trolleybus body around the lateral y-axis, z is the

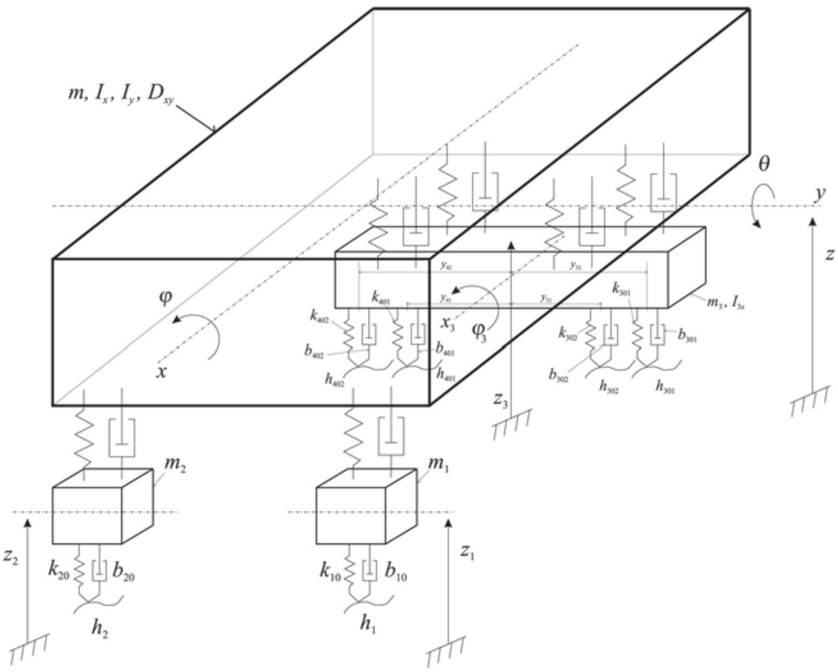


Fig. 2. The scheme of the trolleybus analytical model

vertical displacement of the trolleybus body, z_1 is the vertical displacement of the left front half axle, z_2 is the vertical displacement of the right front half axle, z_3 is the vertical displacement of the rear axle, φ_3 is the angular displacement of the rear axle around the longitudinal axis) and their derivatives with respect to the time, $\mathbf{M}$ is the mass matrix (diagonal elements: I_x – the moment of inertia with respect to x axis of the trolleybus body, I_y – the moment of inertia with respect to y axis of the trolleybus body, m – the mass of the trolleybus body, m_1 – the mass of the left front half axle, m_2 – the mass of the right front half axle, m_3 – the mass of the rear axle, I_{3x} – the moment of inertia with respect to x axis of the rear axle; other elements in case of a symmetric distribution of the sprung mass $m_{ik} = m_{ki} = 0$ for $i \neq k$ and for $i = 1, 2, \ldots, 7$, $k = 1, 2, \ldots, 7$; in case of an asymmetric distribution of the sprung mass $m_{12} = m_{21} = -D_{xy}$, where D_{xy} is the product of inertia with respect to x and y axes of the trolleybus body), $\mathbf{B}$ is the damping matrix, $\mathbf{K}$ is the stiffness matrix (due to the generality all the elements are considered not to be zero), $\mathbf{f}(t) = [f_i(t)]^T$ for $i = 1, 2, \ldots, 7$, is the vector of the generalized forces (kinematic excitation function) [14].

After dividing the individual equations by the respective diagonal element of mass matrix $\mathbf{M}$ (suitable mathematical adjustment due to the solution procedure) and after the Laplace integral transform for the zero initial conditions, i.e. in time $t = 0$ $\mathbf{q}(0) = 0$ and $\dot{\mathbf{q}}(0) = 0$, the system of differential equations is transformed to the system of algebraic equations

$$\mathbf{S} \cdot \bar{\mathbf{q}}(s) = \bar{\mathbf{f}}(s), \tag{2}$$

where s is the parameter of transform, $\bar{\mathbf{q}}(s)$ are the images of the vector of generalized coordinates $\mathbf{q}(t)$ and $\bar{\mathbf{f}}(s)$ are the images of the vector of generalized forces $\mathbf{f}(t)$ divided by the respective diagonal element of mass matrix $\mathbf{M}$.

It holds for the elements of the matrix $\mathbf{S}$:

$$a_{ij} = s^2 + \beta_{ij} \cdot s + \kappa_{ij}, \qquad \text{for } i = j,$$

$$a_{ij} = \delta_{ij} \cdot s^2 + \beta_{ij} \cdot s + \kappa_{ij}, \quad \text{for } i \neq j \text{ and for } i = 1, 2 \text{ and } j = 1, 2,$$

$$a_{ij} = \beta_{ij} \cdot s + \kappa_{ij}, \quad \text{for } i \neq j \text{ and for } i = 1,2 \text{ and } j = 3,4,\ldots,7,$$
$$\text{for } i \neq j \text{ and for } i = 3,4,\ldots,7 \text{ and } j = 1,2,\ldots,7,$$

where $\kappa_{ij} = \frac{k_{ij}}{m_{ii}}$ and $\beta_{ij} = \frac{b_{ij}}{m_{ii}}$ can be calculated from the original elements of stiffness matrix $\mathbf{K}$ and damping matrix $\mathbf{B}$ after division of the equations by the diagonal elements of mass matrix $\mathbf{M}$, $\delta_{12} = -\frac{D_{xy}}{I_x}$ and $\delta_{21} = -\frac{D_{xy}}{I_y}$ are the elements respecting the influence of asymmetric distribution of the mass of the trolleybus body.

For solving the system of algebraic equations (2), i.e. for determining the images of generalized coordinates $\bar{q}_j(s)$, $j = 1, 2, \ldots, 7$, it is possible, due to a small number of equations, to apply the Cramer rule

$$\bar{q}_j(s) = \frac{D_j(s)}{D(s)}, \tag{3}$$

where $D(s)$ is the determinant of matrix $\mathbf{S}$ and $D_j(s)$ is the determinant which originates from determinant $D(s)$ by replacing the j-th column of elements a_{ij} ($i = 1, 2, \ldots, 7$) of determinant $D(s)$ with the column of right sides of the system of linear algebraic equations (2), i.e. with the elements of vector $\bar{\mathbf{f}}(s)$. This method is suitable regarding the process of further solving, i.e. obtaining the vector of generalized coordinates $\mathbf{q}(t)$ by the inverse transform.

For the expansion of determinant $D(s)$ of matrix $\mathbf{S}$ into the form of the polynomial

$$D(s) = \sum_{i=0}^{n=14} A_{n-i} \cdot s^{n-i}, \tag{4}$$

where the polynomial degree n is given by the double of degrees of freedom of the mechanical system (i.e. $n = 14$), it is necessary to determine coefficients A_{n-i} for $i = 1, 2, \ldots, n$ ($A_n = 1$). This operation can be carried out by means of symbolic calculations using the specialized mathematical software.

By evaluating determinant $D_j(s)$ the relation for images $\bar{q}_j(s)$ of function $q_j(t)$ is obtained

$$\bar{q}_j(s) = \sum_{i=1}^{7} (-1)^{j+i} \cdot \bar{f}_i(s) \cdot \frac{D_{ji}(s)}{D(s)}, \text{ for } j = 1, 2, \ldots, 7, \tag{5}$$

where determinant $D_{ji}(s)$ is a subdeterminant of order $n/2-1$ of determinant $D(s)$ corresponding to element a_{ij} of matrix $\mathbf{S}$.

The polynomial corresponding to subdeterminant $D_{ji}(s)$ is determined using the same algorithm as the polynomial (4) if the value of element a_{ij} is changed to $a_{ij} = (-1)^{i+j}$, the other elements $a_{i,j\neq r}$ ($r = 1, 2, \ldots, n$) in the row i are set to zero $a_{i,j\neq r} = 0$ ($r = 1, 2, \ldots, n$) and the other elements $a_{i\neq k,j}$ ($k = 1, 2, \ldots, n$) in the column j are set to zero $a_{i\neq k,j} = 0$ ($k = 1, 2, \ldots, n$), while the values of the other elements $a_{i\neq k,j\neq r}$ ($k = 1, 2, \ldots, n$, $r = 1, 2, \ldots, n$) of determinant $D(s)$ do not change

$$D_{ji}(s) = \sum_{r=0}^{m} d_{ji,m-r} \cdot s^{m-r}, \text{ for } j = 1, 2, \ldots, n/2, \ i = 1, 2, \ldots, n/2, \tag{6}$$

where $m = n - 1$ for $i = j$,
for $i \neq j$ and for $i = 1, 2$ and $j = 1, 2$,
and $m = n - 2$ for $i \neq j$ and for $i = 1, 2$ and $j = 3, 4, \ldots, 7$,
for $i \neq j$ and for $i = 3, 4, \ldots, 7$ and $j = 1, 2, \ldots, 7$,
and coefficients $d_{ji,m-r}$ for $r = 0, 1, \ldots, m$ can be determined using the symbolic calculations.

In order to determine the original $q_j(t)$ of corresponding image $\bar{q}_j(s)$ it is suitable the relation (5) to be transformed to the form of convolution. That is why it is necessary to calculate the zero points s_k of the polynomial of determinant $D(s)$ [14, 18]. In the given case the zero points s_k are supposed to be in the form of the complex conjugate numbers $s_k = \mathrm{Re}\{s_k\} + \mathrm{i} \cdot \mathrm{Im}\{s_k\}$ and $s_{k+1} = \mathrm{Re}\{s_k\} - \mathrm{i} \cdot \mathrm{Im}\{s_k\}$, for $k = 1, 3, \ldots, n-1$ (i is the imaginary unit). By calculating zero points of the polynomial (4) it is possible, using the product of root factors, to put the polynomial in the form of the product of the quadratic polynomials

$$[s - (\mathrm{Re}\{s_i\} + \mathrm{i} \cdot \mathrm{Im}\{s_i\})] \cdot [s - (\mathrm{Re}\{s_i\} - \mathrm{i} \cdot \mathrm{Im}\{s_i\})] = s^2 + p_i \cdot s + r_i, \text{ for } i = 1, 2, \ldots, n/2, \tag{7}$$

where $r_i = (\mathrm{Re}\,\{s_i\})^2 + (\mathrm{Im}\{s_i\})^2$ and $p_i = -2 \cdot \mathrm{Re}\{s_i\}$.

According to (4) it yields

$$D(s) = \sum_{k=0}^{n=14} A_{n-k} \cdot s^{n-k} = \prod_{i=1}^{n/2} \left(s^2 + p_i \cdot s + r_i\right). \tag{8}$$

Then it is possible to transfer the ratio of the determinants in equation (5) to the sum of partial fractions (supposing simple roots) in the form [14]

$$\frac{D_{ji}(s)}{D(s)} = \frac{\sum\limits_{r=0}^{m} d_{ji,m-r} \cdot s^{m-r}}{\prod\limits_{k=1}^{n/2} (s^2 + p_k \cdot s + r_k)} = \frac{\sum\limits_{r=1}^{n/2} \left[(K_{ji,r} \cdot s + L_{ji,r}) \cdot \prod\limits_{\substack{k=1 \\ k \neq r}}^{n/2} (s^2 + p_k \cdot s + r_k) \right]}{\prod\limits_{k=1}^{n/2} (s^2 + p_k \cdot s + r_k)}, \tag{9}$$

where constants $K_{ji,r}$ and $L_{ji,r}$ for $j = 1, 2, \ldots, n/2$, $i = 1, 2, \ldots, n/2$, $r = 1, 2, \ldots, n/2$, can be determined from the condition of the coefficients equality at identical powers of parameter s in numerators of fraction on both sides of equation (9).

The condition of the numerators equality (9) can be expressed by the relation

$$\sum_{r=0}^{m} d_{ji,m-r} \cdot s^{m-r} = \sum_{r=1}^{n/2} \left[(K_{ji,r} \cdot s + L_{ji,r}) \cdot \frac{\prod\limits_{k=1}^{n/2} (s^2 + p_k \cdot s + r_k)}{s^2 + p_r \cdot s + r_r} \right], \tag{10}$$

where m ($m = n-1$ or $m = n-2$) is the order of the polynomial of determinant $D_{ji}(s)$, j is the designation of the component of vector of the images of generalized coordinates $\bar{q}_j(s)$ ($j = 1, 2, \ldots, n/2$) and i is the designation of the component of vector of the images of the generalized forces (divided by the respective diagonal element of mass matrix $\mathbf{M}$) ($i = 1, 2, \ldots, n/2$).

As the denominator of fraction on the right side of equation (10) is the divisor of the numerator (i.e. division remainder is equal zero), this fraction can be, regarding the relation (8), modified to the form [14, 18]

$$\frac{\prod\limits_{k=1}^{n/2} (s^2 + p_k \cdot s + r_k)}{s^2 + p_i \cdot s + r_i} = \frac{\sum\limits_{k=0}^{n} A_{n-k} \cdot s^{n-k}}{s^2 + p_i \cdot s + r_i} = \sum_{k=2}^{n} t_{i,n-k} \cdot s^{n-k}, \text{ for } i = 1, 2, \ldots, n/2, \tag{11}$$

where $t_{i,n-1} = 0,$
$t_{i,n-2} = A_n = 1,$
$t_{i,n-k} = A_{n-k+2} - p_i \cdot t_{i,n-k+1} - r_i \cdot t_{i,n-k+2},$ for $k = 3, 4, \ldots, n,\ i = 1, 2, \ldots, n/2.$

Then equation (10) can be put in the form

$$\sum_{r=0}^{m} d_{ji,m-r} \cdot s^{m-r} = \sum_{r=1}^{n/2} \left[(K_{ji,r} \cdot s + L_{ji,r}) \cdot \sum_{k=2}^{n} t_{r,n-k} \cdot s^{n-k} \right], \tag{12}$$

for $j = 1, 2, \ldots, n/2$, $i = 1, 2, \ldots, n/2$.

After performing the multiplication of the polynomials on the right side of equation (12) and comparing the coefficients of identical powers of parameter s on both sides of equation (12) a system of n algebraic equations for unknown coefficients $K_{ji,r}$ and $L_{ji,r}$ for $j = 1, 2, \ldots, n/2$, $i = 1, 2, \ldots, n/2$ and $r = 1, 2, \ldots, n/2$ is obtained

$$\sum_{r=1}^{n/2} (K_{ji,r} \cdot t_{r,n-k-1} + L_{ji,r} \cdot t_{r,n-k}) = d_{ji,n-k}, \text{ for } k = 1, 2, \ldots, n, \tag{13}$$

where for $k = n$ coefficients $t_{r,n-k-1} = t_{r,-1} = 0$, $r = 1, 2, \ldots, n/2$, represent the division remainders of equation (11) – see [14] or [18].

By analytical solving the system of algebraic equations (13) – see [14] – unknown coefficients $K_{ji,r}$ and $L_{ji,r}$ are determined and equation (9) (using relation (10)) can be put in the form

$$\frac{D_{ji}(s)}{D(s)} = \sum_{k=1}^{n/2} \frac{K_{ji,k} \cdot s + L_{ji,k}}{s^2 + p_k \cdot s + r_k}, \text{ for } j = 1, 2, \ldots, n/2,\ i = 1, 2, \ldots, n/2. \tag{14}$$

By means of this relation equation (5) for the calculation of the image of generalized coordinates $\bar{q}_j(s)$, for $j = 1, 2, \ldots, n/2$, can be modified into the form

$$\bar{q}_j(s) = \sum_{i=1}^{n/2} (-1)^{j+i} \cdot \bar{f}_i(s) \cdot \sum_{k=1}^{n/2} \frac{K_{ji,k} \cdot s + L_{ji,k}}{s^2 + p_k \cdot s + r_k}. \tag{15}$$

The denominator of the fraction of equation (15) can be modified to the form

$$s^2 + p_k \cdot s + r_k = (s + \beta_k)^2 + \Omega_k^2, \text{ for } k = 1, 2, \ldots, n/2, \tag{16}$$

where $\Omega_k^2 = \omega_k^2 - \beta_k^2$ is the damped natural frequency, $\omega_k^2 = r_k$ is the undamped natural frequency and $\beta_k = -\frac{p_k}{2}$ is the coefficient of linear viscous damping.

Using formula (16) equation (15) can be written in the form

$$\bar{q}_j(s) = \sum_{i=1}^{n/2} (-1)^{j+i} \cdot \bar{f}_i(s) \cdot \sum_{k=1}^{n/2} \frac{K_{ji,k} \cdot s + L_{ji,k}}{(s + \beta_k)^2 + \Omega_k^2}, \text{ for } j = 1, 2, \ldots, 7. \tag{17}$$

Further modification may result in the final relation for image $\bar{q}_j(s)$ for $j = 1, 2, \ldots, 7$

$$\begin{aligned} \bar{q}_j(s) \;=\; & \sum_{i=1}^{n/2} (-1)^{j+i} \cdot \bar{f}_i(s) \cdot \sum_{k=1}^{n/2} \left[K_{ji,k} \cdot \frac{s + \beta_k}{(s + \beta_k)^2 + \Omega_k^2} + \right. \\ & \left. + \frac{L_{ji,k} - \beta_k \cdot K_{ji,k}}{\Omega_k} \cdot \frac{\Omega_k}{(s + \beta_k)^2 + \Omega_k^2} \right]. \end{aligned} \tag{18}$$

After the inverse transform of the relation (18), the function of generalized coordinate $q_j(t)$, for $j = 1, 2, \ldots, 7$, in the form of the sum of convolution integrals is obtained

$$\begin{aligned} q_j(t) \;=\; & \sum_{i=1}^{n/2} (-1)^{j+i} \cdot \sum_{k=1}^{n/2} \left\{ K_{ji,k} \cdot \int_0^t \frac{f_i(\tau)}{m_{ii}} \cdot \mathrm{e}^{-\beta_k \cdot (t-\tau)} \cdot \cos\left[\Omega_k \cdot (t-\tau)\right] \cdot \mathrm{d}\tau + \right. \\ & \left. + \frac{L_{ji,k} - \beta_k \cdot K_{ji,k}}{\Omega_k} \cdot \int_0^t \frac{f_i(\tau)}{m_{ii}} \cdot \mathrm{e}^{-\beta_k \cdot (t-\tau)} \cdot \sin\left[\Omega_k \cdot (t-\tau)\right] \cdot \mathrm{d}\tau \right\}, \end{aligned} \tag{19}$$

where the elements of the excitation force vector are generally given by the relations

$$\begin{aligned} f_1(t) &= 0, \\ f_2(t) &= 0, \\ f_3(t) &= 0, \\ f_4(t) &= k_{10} \cdot h_1(t) + b_{10} \cdot \dot{h}_1(t), \\ f_5(t) &= k_{20} \cdot h_2(t) + b_{20} \cdot \dot{h}_2(t), \\ f_6(t) &= k_{301} \cdot h_{31}(t) + k_{302} \cdot h_{32}(t) + k_{401} \cdot h_{41}(t) + k_{402} \cdot h_{42}(t) + \\ & \quad + b_{301} \cdot \dot{h}_{31}(t) + b_{302} \cdot \dot{h}_{32}(t) + b_{401} \cdot \dot{h}_{41}(t) + b_{402} \cdot \dot{h}_{42}(t), \\ f_7(t) &= -k_{301} \cdot h_{31}(t) \cdot y_{31} - k_{302} \cdot h_{32}(t) \cdot y_{32} + k_{401} \cdot h_{41}(t) \cdot y_{41} + k_{402} \cdot h_{42}(t) \cdot y_{42} + \\ & \quad - b_{301} \cdot \dot{h}_{31}(t) \cdot y_{31} - b_{302} \cdot \dot{h}_{32}(t) \cdot y_{32} + b_{401} \cdot \dot{h}_{41}(t) \cdot y_{41} + b_{402} \cdot \dot{h}_{42}(t) \cdot y_{42}, \end{aligned} \tag{20}$$

where $h_1(t)$, $h_2(t)$, $h_{31}(t)$, $h_{32}(t)$, $h_{41}(t)$, $h_{42}(t)$ are the functions describing the shape of the road surface unevenness under the tires, $\dot{h}_1(t)$, $\dot{h}_2(t)$, $\dot{h}_{31}(t)$, $\dot{h}_{32}(t)$, $\dot{h}_{41}(t)$, $\dot{h}_{42}(t)$ are the first-order derivatives of the functions $h_1(t)$, $h_2(t)$, $h_{31}(t)$, $h_{32}(t)$, $h_{41}(t)$, $h_{42}(t)$, k_{10}, k_{20}, k_{301}, k_{302}, k_{401}, k_{402} are the (linear) radial stiffnesses of the tires, b_{10}, b_{20}, b_{301}, b_{302}, b_{401}, b_{402} are the (linear) coefficients of radial damping of the tires and y_{31}, y_{32}, y_{41}, y_{42} are the lateral coordinates of the centres of mass of the rear tires. Subscripts 1 and 10 belong to the left front tire, subscripts 2 and 20 to the right front tire, subscripts 31 and 301 to the left rear outside tire, subscripts 32 and 302 to the left rear inside tire, subscripts 41 and 401 to the right rear inside tire and subscripts 42 and 402 to the right rear outside tire (see fig. 2). See [13] or [14] for more details.

3. Numerical model

The multibody model of the ŠKODA 21 Tr low-floor trolleybus is created in the *alaska 2.3* simulation tool. As it is the first comparison of the results of the simulations performed with the analytical model and with the numerical model there is not utilized the most complex multibody model (formed by 35 rigid bodies and two superelements mutually coupled by 52 joints – e.g. [7, 8]; see fig. 3), but the simplest multibody model created in the *alaska 2.3* simulation tool (e.g. [7]; see fig. 4). The multibody model of the trolleybus is formed by 29 rigid bodies mutually coupled by 29 kinematic joints. The rigid bodies correspond generally to the vehicle individual structural parts. The number of degrees of freedom in kinematic joints is 47.

The rigid bodies are defined by inertia properties (mass, centre of mass coordinates and moments of inertia). The air springs and the hydraulic shock absorbers in the axles suspension and the bushings in the places of mounting some trolleybus structural parts are modelled by connecting the corresponding bodies by nonlinear spring-damper elements. When simulating driving on the uneven road surface the contact point model of tires is used in the multibody model; radial stiffness and radial damping properties of the tires are modelled by nonlinear spring-damper elements considering the possibility of bounce of the tire from the road surface [3, 8, 9].

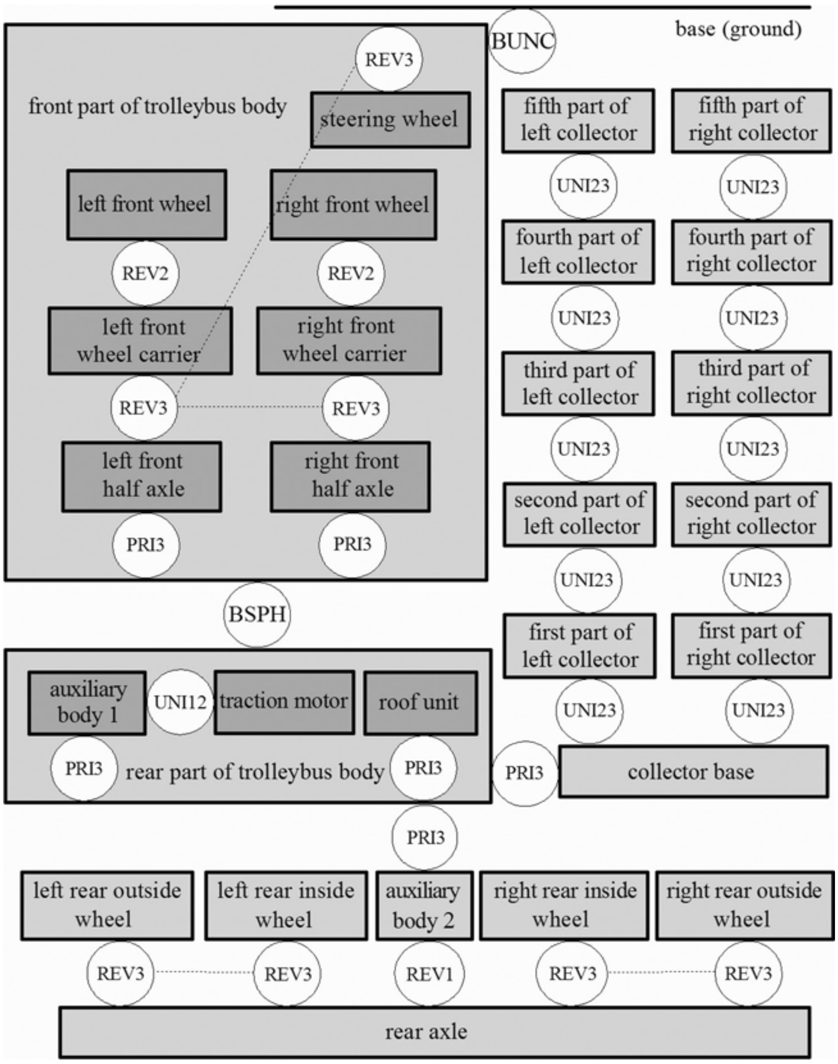


Fig. 3. Kinematic scheme of the multibody model of the ŠKODA 21 Tr trolleybus

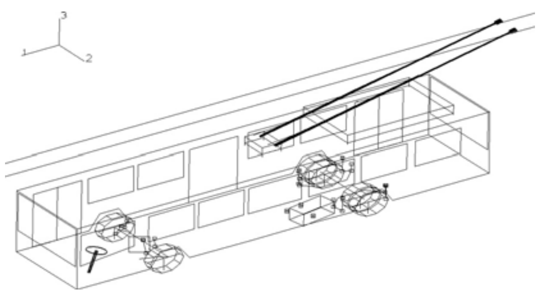


Fig. 4. Visualization of the ŠKODA 21 Tr trolleybus multibody model in the *alaska 2.3* simulation tool

The kinematic scheme of the multibody model of the ŠKODA 21 Tr low-floor trolleybus is shown in fig. 3, where circles represent kinematic joints (BUNC – unconstrained, BSPH – spherical, UNI12 – universal around axes 1 and 2, UNI23 – universal around axes 2 and 3, PRI3 – prismatic in axis 3 direction, REV1 – revolute around axis 1, REV2 – revolute around axis 2, REV3 – revolute around axis 3; axes of the coordinate system are considered according to fig. 4) and quadrangles represent rigid bodies.

4. Simulations results

In order to illustrate the vertical dynamic response calculated by means of the analytical approach and numerical simulation with the trolleybus virtual models, the driving on the artificially created test track according to the ŠKODA VÝZKUM methodology was chosen (e.g. [2, 3, 7, 8, 9, 10]). The test track consisted of three standardized artificial obstacles (in compliance with the Czech Standard ČSN 30 0560 Obstacle II: $h = 60$ mm, $R = 551$ mm, $d = 500$ mm) spaced out on the smooth road surface 20 meters one after another. The first obstacle was run over only with right wheels, the second one with both and the third one only with left wheels (see fig. 5). Results of the drive at the trolleybus models' speed 40 km/h (the usual trolleybus speed according to the ŠKODA VÝZKUM methodology at the driving on the artificially created test track) are shown.

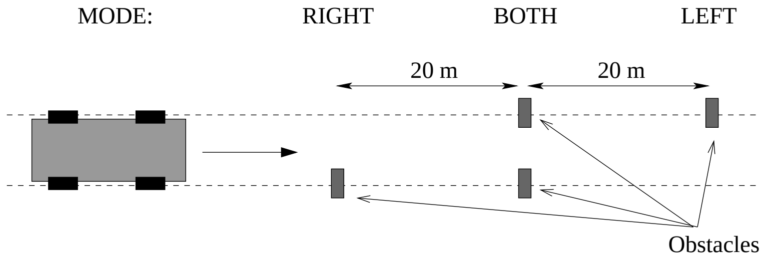
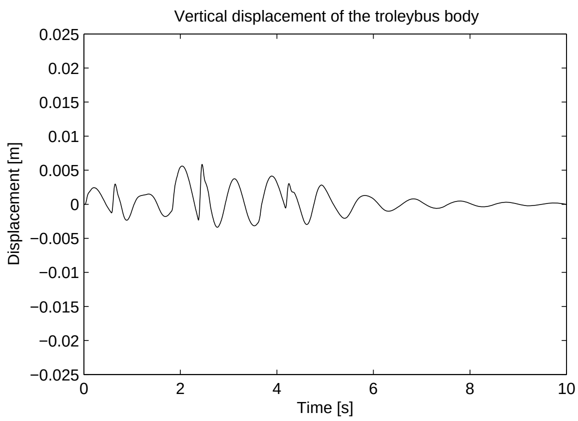


Fig. 5. Visualization of the ŠKODA 21 Tr trolleybus multibody model in the *alaska 2.3* simulation tool

In the course of the test drives simulations time histories of the vertical displacements (see fig. 2) of the trolleybus body z (fig. 6), the left front half axle z_1 (fig. 7), the right front half axle z_2 (fig. 8) and the rear axle z_3 (fig. 9) were monitored (among others).

On the basis of the monitored quantities given in figs 6 to 9 it is generally possible to say that in the results obtained using nonlinear numerical model extreme values of the time histories of the vertical displacements are higher, the decay of dynamic responses of the unsprung masses to the kinematic excitation of the wheels is slower and the decay of dynamic response of the sprung mass to the kinematic excitation of the wheels is faster than in the results obtained using the linear model. But the results are not as different as it was supposed.



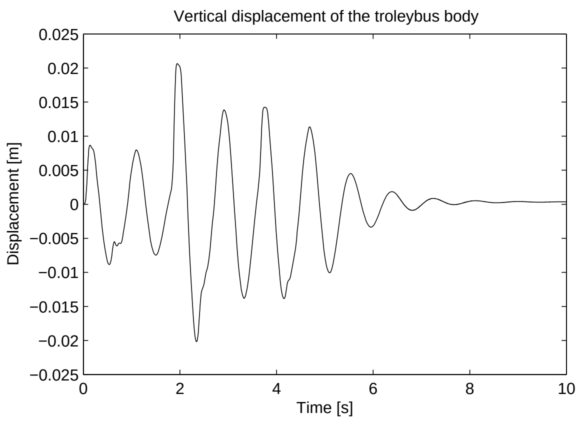


Fig. 6. Time histories of the vertical displacement of the trolleybus body: of the linear model of the trolleybus – left, of the nonlinear model of the trolleybus – right

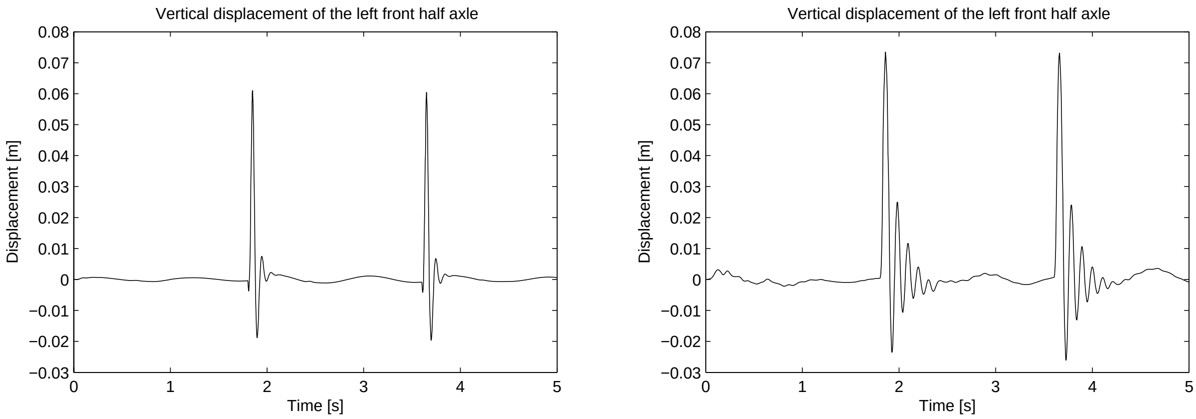


Fig. 7. Time histories of the vertical displacement of the left front half axle: of the linear model of the trolleybus – left, of the nonlinear model of the trolleybus – right

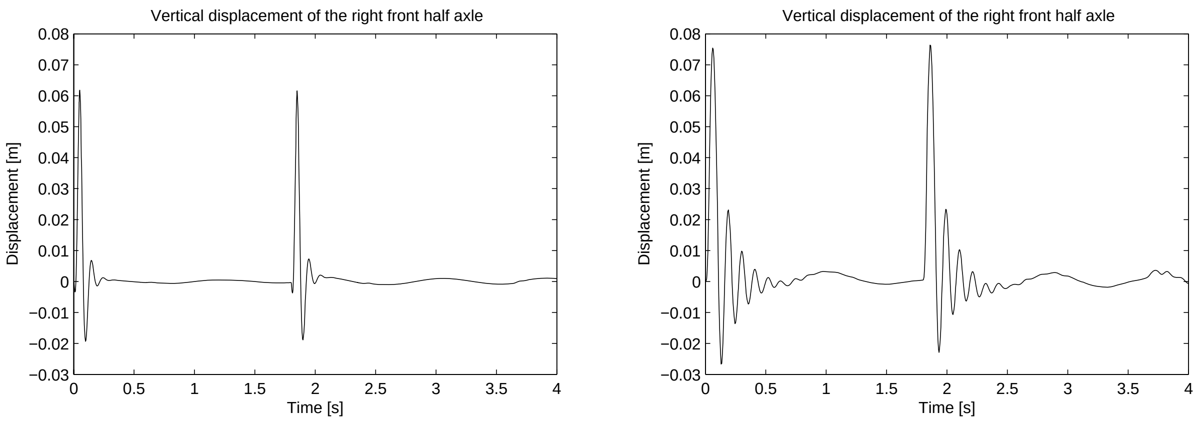


Fig. 8. Time histories of the vertical displacement of the right front half axle: of the linear model of the trolleybus – left, of the nonlinear model of the trolleybus – right

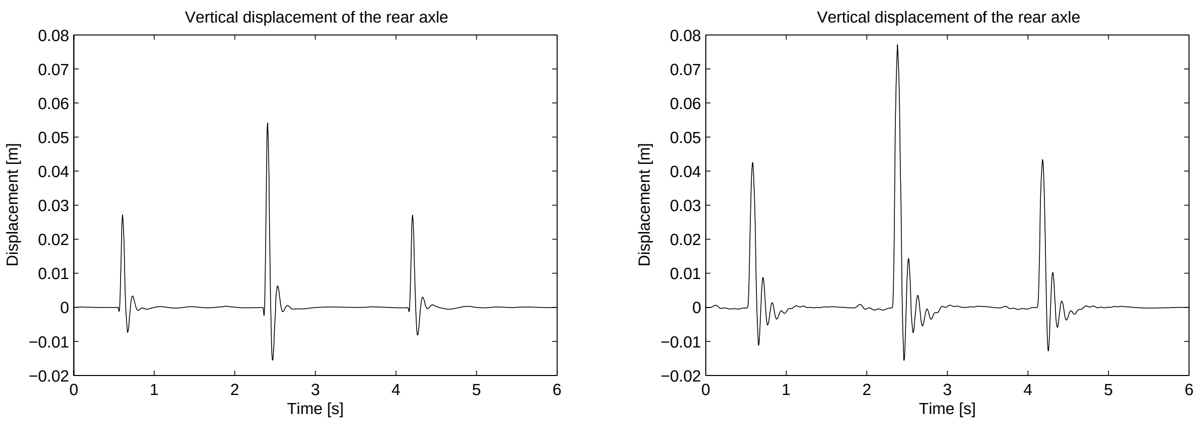


Fig. 9. Time histories of the vertical displacement of the rear axle: of the linear model of the trolleybus – left, of the nonlinear model of the trolleybus – right

On the basis of the test simulations with the trolleybus model created in the *alaska* simulation tool the following facts were stated: the consideration of the only linear force-velocity characteristics of the shock absorbers in the linear model of the trolleybus has the greatest influence on the extreme values of the vertical displacement of the sprung mass, the consideration of the only linear radial stiffness of the tires in the linear model has the greatest influence on the extreme values of the displacements of the unsprung masses, the consideration of the only linear force-deformation characteristics of the air springs and (at the trolleybus models speed 40 km/h) the nonconsideration of the possibility of the tire bounce from the road surface in the linear model have the greatest influence on the decay of the dynamic response of the trolleybus body and the consideration of the only linear radial stiffness of the tires in the linear model has the greatest influence on the decay of the responses of the unsprung masses.

5. Conclusion

Two virtual models of the ŠKODA 21 Tr low-floor trolleybus intended for the investigation of vertical dynamic properties during the simulation of driving along the uneven road surface are presented in the article. The first model and its dynamic response are based on the analytical solution, the second one is based on the multibody modelling and the numerical simulations. Both models have various advantages and together they are complex tools for the vehicle vertical dynamics investigation. The linear analytical model is suitable for the fast and accurate analysis and can be employed mainly for the optimization or the control tasks. The complex multibody model can be used in further steps for a detailed manoeuvre analysis and a particular structural elements evaluation. Differences between the results obtained using both models are discussed.

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