

# MHD mixed flow of unsteady convection with radiation over a vertical porous plate: Lie group symmetry analysis

G. R. Rajput<sup>a,\*</sup>, V. S. Patil<sup>b</sup>, B. P. Jadhav<sup>a</sup>

<sup>a</sup>Department of Applied Mathematics, SVKMs, NMIMS, Mukesh Patel School of Technology Management and Engineering, Shirpur campus, Shirpur-425405, India

<sup>b</sup>Department of Mathematics, Govt. College of Engineering, Karad-415124, India

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## Abstract

In this paper, we study the magnetohydrodynamic (MHD) mixed unsteady flow over a vertical porous plate. The system of non-linear partial differential equations governing the physical model is transformed into a system of non-linear ordinary differential equations via Lie group analysis. Using the Runge-Kutta fourth order method along with shooting technique the numerical analysis is carried out to study the effect of associated parameters on the velocity, temperature and concentration distribution. Computed results for the velocity, temperature and concentration distribution are discussed graphically.

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*Keywords:* MHD, porous media, symmetry analysis

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## 1. Introduction

In recent years, the flow of fluid through porous media has attracted extensive attention of many researchers due to their vast applications in material sciences and engineering such as Petroleum industries, seepage of water in riverbeds, filtration and purification of chemical processes and many others. The mixed convection flow come across many industrial applications such as cooling of nuclear reactors, cooling of sophisticated electronic devices, solar panel exposed to wind currents etc. In the literature a comprehensive survey of convective heat transfer mechanism through porous media was first reported by Nield et al. [7]. A systematic simulation for the mass transfer effect on the two-dimensional unsteady MHD free convective flow past a vertical porous plate with variable suction were investigated by Ramana Kumari et al. [13]. Kandasamy et al. [6] reported the effects of thermal and mass transfer diffusion in MHD mixed convective flow over a porous wedge with heat radiation and considering suction/injection. The hydromagnetic unsteady mixed convection flow past porous plate sop in porous media was studied by Sharma et al. [16]. Uddin et al. [26] analysed unsteady MHD convective flow of heat and mass transfer past a vertical surface. Unsteady hydromagnetic mixed convection flow of heat and mass transfer in the stagnation region with a complex wall condition were studied by Chamka et al. [2]. Elbashbeshy and Aldawody [4] explained the impact of magnetism and radiation on unsteady mixed convection flow and heat transfer past a porous medium with generation/absorption of heat.

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\*Corresponding author. E-mail: g.rajput7@gmail.com.  
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### Nomenclature

$Gr$ Grashof number	Greek symbols:
$Gm$ modified Grashof number	$\beta$ coefficient of volume expansion for heat transfer ( $K^{-1}$ )
$x, y$ dimensionless coordinates (m)	$\beta^*$ coefficient of volume expansion for mass transfer ( $m^3 \cdot kg^{-1}$ )
$t'$ time in $x', y'$ coordinate system (s)	$\sigma$ electrical conductivity of fluid ( $S \cdot m^{-1}$ )
$t$ time in dimensionless coordinates (s)	$\nu$ kinematic viscosity ( $m^2 \cdot s^{-1}$ )
$u$ dimensionless velocity component in $x$ -direction (s)	$\theta$ non dimensional temperature (K)
$T$ temperature of fluid (K)	$\phi$ concentration of fluid ( $mol \cdot m^{-3}$ )
$T_w$ temperature of fluid near plate (K)	$\rho$ density of the fluid ( $Kg \cdot m^{-3}$ )
$C$ concentration of the fluid ( $mol \cdot m^{-3}$ )	$\sigma_s$ Stefan-Boltzmann constant
$C_p$ specific heat at constant pressure	
$g$ gravity acceleration ( $m/s^2$ )	
$Pr$ Prandtl number	
$D_m$ mass diffusivity ( $m^2 \cdot s^{-1}$ )	
$Sc$ Schmidt number	
$Fs$ local Forchheimer number	
$M$ magnetic number	
$Ec$ Eckert number	
$Kr$ chemical reaction parameter	
$Da$ local Darcy number	
$Ke$ mean absorption coefficient	
$C_w$ concentration of the fluid near plate ( $mol \cdot m^{-3}$ )	

Several authors have studied the MHD free convection flow of heat and mass transfer through porous media subject to different boundary conditions. Raju et al. [9] have investigated unsteady MHD flow past an exponentially accelerated isothermal vertical plate in the existence of heat absorption and variable temperature by using Laplace transform. Sheri et al. [23] studied the transient MHD free convection flow past an infinite vertical plate fixed in a porous medium with viscous dissipation. Raju et al. [10] applied FEM to unsteady MHD free convection flow past a vertically inclined porous plate. Raju et al. [11] studied numerically MHD free convection Couette flow with the effect of Soret and Dufour numbers. Raju et al. [8] has investigated combined effects of thermal-diffusion and diffusion-thermo over free convection flow over an infinite porous plate with magnetic field and chemical reaction using FEM. Further Srinivasa et al. [25] studied numerically the effects of chemical reaction on unsteady MHD flow past an exponentially accelerated plate in existence of heat absorption and variable temperature using FEM and LTT. The MHD boundary layer flow of nanofluid and heat transfer over nonlinear stretching sheet in the existence of chemical reaction is analysed by Ramya et al. [14]. Das [3] obtained a complete solution of free convection flow over vertical plate with heat and mass transfer with thermal radiation. Raju et al. [12] investigated effects of thermal radiation and heat source on an unsteady MHD free convective flow past an infinite vertical plate in porous medium in presence of thermal diffusion. In recent decades various research articles are published about nanofluid heat transfer enhancement [17–22].

The study reported here in considers an unsteady mixed convection flow with radiation over a vertical porous plate. The system of non-linear partial differential equations governing the physical model has been transformed into non-linear ordinary differential equations in similarity variables via the Lie group analysis and solved numerically by a Runge-Kutta fourth order method along with shooting technique. The flow phenomenon has been characterized by associated parameters and their effects on the velocity, temperature and concentration profiles have been analysed and the results obtained are discussed graphically. It is hoped that the present investigation will serve as an effective complement to the previous studies.

## 2. Problem formulation

Consider an incompressible, unsteady flow of an electrically conducting viscous fluid over vertical porous plate. We considered infinite plate at  $x$ -axis, which is in parallel with free stream velocity and the  $y$ -axis is taken normal to the plate.  $B_0$  is the magnetic induction which is employed in the flow direction. Initially the plate and fluid follows the equal temperature  $T_\infty$  in a stable condition with concentration  $C_\infty$  at all points. The plate starts moving impulsively for  $t > 0$  with a constant velocity  $U_0$  and its temperature and concentration boost to  $T_w$  and  $C_w$  respectively. All the fluid physical properties are considered to be constant except the influence of the body force term. The flow geometry is described in Fig. 1.

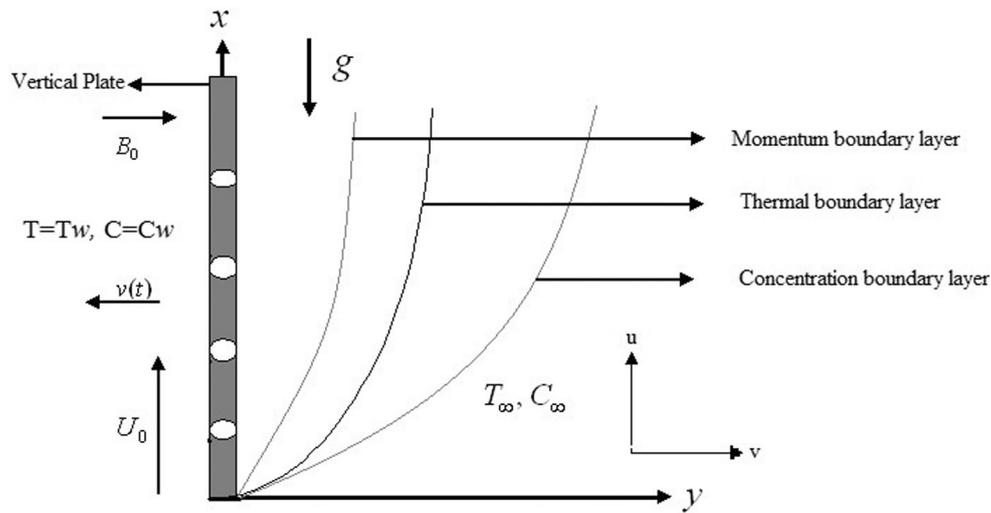


Fig. 1. Flow Geometry

With an assumptions that the Boussinesq and boundary layer approximations hold good, and accepting the Darcy-Forchheimer model, the system of equations which models the flow is given by

$$\frac{\partial v}{\partial y} = 0, \quad (1)$$

$$\frac{\partial u}{\partial t'} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{K} u - \frac{b}{K} u^2, \quad (2)$$

$$\frac{\partial T}{\partial t'} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} + \frac{v}{c_p} \left( \frac{\partial u}{\partial y} \right)^2, \quad (3)$$

$$\frac{\partial C}{\partial t} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - k_r(C - C_\infty). \quad (4)$$

By using the Rosseland approximation [15, 24], the radiative heat flux  $q_r$  is given by

$$q_r = -\frac{4\sigma_s}{3k_e} \frac{\partial T^4}{\partial y}. \quad (5)$$

Here  $u$  and  $v$  are the Darcian velocity components along the  $x$  and  $y$  directions, respectively. The symbol  $\alpha$  stands for the thermal diffusivity,  $\rho$  is the density of fluid,  $\beta$  is the volume expansion coefficient,  $\nu$  is the kinematic viscosity,  $\beta^*$  is the coefficient of volumetric expansion,  $K$  is the Darcy permeability,  $b$  is the empirical constant,  $T$  is the fluid temperature inside the thermal

boundary layer and  $T_\infty$  is the temperature of fluid over the free stream while  $C$  and  $C_\infty$  are the corresponding concentrations, respectively. Also  $\sigma_s$  is the Stefan-Boltzmann constant and  $k_e$  the mean absorption coefficient.  $D_m$  is the mass diffusivity coefficient,  $k_r$  is chemical reaction parameter.

Therefore, the no slip boundary conditions at the surface of the plate are given by

$$\begin{aligned} u = U_0, \quad T = T_w, \quad C = C_w \quad \text{at} \quad y = 0, \\ u = 0, \quad T = T_\infty, \quad C = C_\infty \quad \text{as} \quad y \rightarrow \infty. \end{aligned} \tag{6}$$

### 2.1. Reduction to non-dimensional form

$$\begin{aligned} U = \frac{u}{u_0}, \quad Y = \frac{u_0 y}{\nu}, \quad t = \frac{u_0 t'}{\nu}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty}, \\ Gr = \frac{\nu \beta g (T_w - T_\infty)}{u_0^3}, \quad Gm = \frac{\nu \beta^* g (C_w - C_\infty)}{u_0^3}, \quad Pr = \frac{\nu}{\alpha}, \quad Sc = \frac{\nu}{Dm}, \\ M = \frac{\sigma B_0^2 \nu}{\rho u_0^2}, \quad Ec = \frac{u_0^2}{c_p (T_w - T_\infty)}, \quad R = \frac{K_T K^*}{4 \sigma_r T_\infty^3}, \\ Da = \frac{K u_0^2}{\nu^2}, \quad Fs = \frac{b u_0}{\nu}, \quad K^2 r = \frac{k_r \nu}{u_0^2}. \end{aligned} \tag{7}$$

By using the dimensionless variables and constants (7), the system of equations (2)–(4) is reduced to

$$\frac{\partial U}{\partial t} - \frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} + Gr\theta + Gm\phi - MU - \frac{1}{Da}U - \frac{Fs}{Da}U^2, \tag{8}$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial Y} = \frac{1}{Pr} \left[ 1 + \frac{4}{3R} \right] \frac{\partial^2 \theta}{\partial Y^2} + Ec \left( \frac{\partial U}{\partial Y} \right)^2, \tag{9}$$

$$\frac{\partial \phi}{\partial t} - \frac{\partial \phi}{\partial Y} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial Y^2} - K^2 r \phi. \tag{10}$$

The boundary conditions are

$$\begin{aligned} U = 1, \quad \theta = 1, \quad \phi = 1 \quad \text{at} \quad Y = 0, \\ U = 0, \quad \theta = 0, \quad \phi = 0 \quad \text{as} \quad Y \rightarrow \infty, \end{aligned} \tag{11}$$

where  $Gr$  is the thermal Grashof number,  $Gm$  is the solutal Grashof number,  $M$  is the magnetic parameter,  $Da$  is the local Darcy number,  $Fs$  is the local Forchheimer number,  $Pr$  is the Prandtl number,  $R$  is the thermal radiation,  $Ec$  is the Eckert number,  $Sc$  is the Schmidt number and  $Kr$  is a chemical reaction parameter. We select  $Pr = 0.71$  which corresponds to 20 °C in the air and 1 atmospheric pressure of  $Sc = 0.6$ . We have chosen the values of  $Gr$  and  $Gm$  larger due to the convection problem that causes cooling in the plate.

### 3. Lie group symmetry analysis

Most of the physical problems are modeled either in terms of nonlinear ordinary or partial differential equations. A suitable method for dealing such types of nonlinear equations is provided by the Lie group analysis. The Lie group analysis proposes an accurate mathematical formulation of spontaneous ideas of symmetry and provides constructive methods for solving non-linear

differential equations analytically. The Lie group analysis affirms symmetries of differential equations that cannot be seen otherwise. Various physical phenomena can be explored using the Lie symmetries to ascertain various group invariant solutions and conservation laws that cater significant physical insight into the problem. Nowadays the Lie group analysis is extensively used both in applying classical and developing new methods. An exhaustive collection of the results of applications of the Lie group analysis to applied mathematics is given in three volumes of [5]. In this section our aim is to impart the use of the group theoretic approach. Let us consider a one-parameter Lie group of infinitesimal transformations as

$$\begin{aligned} t^* &= t + \varepsilon \xi^T(t, Y, U, \theta, \phi) + O(\varepsilon^2), \\ Y^* &= Y + \varepsilon \xi^Y(t, Y, U, \theta, \phi) + O(\varepsilon^2), \\ U^* &= U + \varepsilon \eta^U(t, Y, U, \theta, \phi) + O(\varepsilon^2), \\ \theta^* &= \theta + \varepsilon \eta^\theta(t, Y, U, \theta, \phi) + O(\varepsilon^2), \\ \phi^* &= \phi + \varepsilon \eta^\phi(t, Y, U, \theta, \phi) + O(\varepsilon^2), \end{aligned} \tag{12}$$

where small group parameter  $\varepsilon \ll 1$ .

Consider the associated Lie algebra as

$$\begin{aligned} \chi &= \xi^t(t, Y, U, \theta, \phi) \frac{\partial}{\partial x} + \xi^Y(t, Y, U, \theta, \phi) \frac{\partial}{\partial y} + \eta^Y(t, Y, U, \theta, \phi) \frac{\partial}{\partial u} + \\ &\eta^\theta(t, Y, U, \theta, \phi) \frac{\partial}{\partial \theta} + \eta^\phi(t, Y, U, \theta, \phi) \frac{\partial}{\partial \phi}. \end{aligned} \tag{13}$$

The action of  $\chi$  is extended to all derivatives appearing in the equations (8)–(11) and can be written as

$$\begin{aligned} \chi^{[2]} &= \chi + \eta_t^U \frac{\partial}{\partial u_t} + \eta_Y^U \frac{\partial}{\partial u_Y} + \eta_t^\theta \frac{\partial}{\partial \theta_t} + \eta_Y^\theta \frac{\partial}{\partial \theta_Y} + \eta_t^\phi \frac{\partial}{\partial \phi_t} + \eta_Y^\phi \frac{\partial}{\partial \phi_Y} + \\ &\eta_{tY}^U \frac{\partial}{\partial u_{tY}} + \eta_{tY}^\theta \frac{\partial}{\partial \theta_{tY}} + \eta_{tY}^\phi \frac{\partial}{\partial \phi_{tY}}, \end{aligned} \tag{14}$$

where

$$\begin{aligned} \eta_t^Y &= D_x(\eta^U) - U_t D_t(\xi^t) - U_Y D_t(\xi^Y), \\ \eta_Y^U &= D_Y(\eta^U) - U_t D_t(\xi^t) - U_Y D_t(\xi^Y), \\ \eta_t^\theta &= D_t(\eta^\theta) - \theta_t D_t(\xi^t) - \theta_Y D_t(\xi^Y), \\ \eta_Y^\theta &= D_Y(\eta^\theta) - \theta_t D_t(\xi^t) - \theta_Y D_t(\xi^Y), \\ \eta_t^\phi &= D_t(\eta^\phi) - \phi_t D_t(\xi^t) - \phi_Y D_t(\xi^Y), \\ \eta_Y^\phi &= D_Y(\eta^\phi) - \phi_t D_t(\xi^t) - \phi_Y D_t(\xi^Y), \\ \eta_{tY}^U &= D_Y(\eta_Y^U) - U_{tY} D_Y(\xi^t) - U_{YY} D_Y(\xi^Y), \\ \eta_{tY}^\theta &= D_Y(\eta_Y^\theta) - \theta_{tY} D_Y(\xi^t) - \theta_{YY} D_Y(\xi^Y), \\ \eta_{tY}^\phi &= D_Y(\eta_Y^\phi) - \phi_{tY} D_Y(\xi^t) - \phi_{YY} D_Y(\xi^Y), \end{aligned} \tag{15}$$

where  $D_t$  and  $D_Y$  are total differentiation operators w.r.t.  $x$  and  $y$ , respectively.

The operator  $\chi$  is the point symmetry of the equations (8)–(11), if

$$\chi^{[2]}(\Delta_j)|_{\Delta_j} = 0, \quad j = 1, 2, 3, 4, \tag{16}$$

where  $\Delta_j$  are given by equations (8)–(11), respectively.

As the coefficients of  $\chi$  do not contains differentiations, hence we separate the equation (16) w.r.t. the derivatives and model of linear homogeneous partial differential equation (15) yields

$$\begin{aligned} \xi_\phi^t &= \xi_\theta^t = \xi_U^t = \xi_t^t = \xi_Y^t = 0, \\ \xi_\phi^Y &= \xi_\theta^Y = \xi_U^Y = \xi_Y^Y = 0, \\ \eta^\theta &= \eta^\phi = \eta^U = 0, \\ U\xi_t^Y &= 0, \end{aligned} \tag{17}$$

where the suffixes denote the partial derivatives.

The required form of the infinitesimals are

$$\begin{aligned} \xi^t &= c, \\ \xi^Y &= g(t), \\ \eta^U &= 0, \\ \eta^\theta &= 0, \\ \eta^\phi &= 0, \end{aligned} \tag{18}$$

where  $c$  is a constant of translation transformation and  $g(t)$  is an arbitrary functions.

Considering the boundary conditions (11), equation (18) becomes

$$\begin{aligned} \xi^t &= c, \\ \xi^Y &= 0, \\ \eta^U &= 0, \\ \eta^\theta &= 0, \\ \eta^\phi &= 0. \end{aligned} \tag{19}$$

### 3.1. Reductions to ODEs

Using the infinitesimals reported by Bluman and Kumai [1], the characteristic equations are given by

$$\frac{dt}{c} = \frac{dY}{0} = \frac{dU}{0} = \frac{d\theta}{0} = \frac{d\phi}{0}. \tag{20}$$

This yields similarity variables as

$$\eta = Y, \quad U = f_1(\eta), \quad \theta = f_2(\eta), \quad \phi = f_3(\eta). \tag{21}$$

Substituting similarity variables (21) into basic equations (8)–(11), we obtain the similarity equations

$$f_1'' + f_1' + Grf_2 + Gmf_3 - Mf_1 - \frac{1}{Da}f_1 - \frac{Fs}{Da}f_1^2 = 0, \tag{22}$$

$$f_2'' \frac{1}{Pr} \left( 1 + \frac{4}{3R} \right) + f_2' + Ecf_1'^2 = 0, \tag{23}$$

$$\frac{1}{Sc}f_3'' + f_3' - K_r^2f_3 = 0 \tag{24}$$

with the transformed boundary conditions

$$\begin{aligned} f_1(0) &= 1, \quad f_2(0) = 1, \quad f_3(0) = 1, \\ f_1(\infty) &= f_2(\infty) = f_3(\infty) = 0. \end{aligned} \tag{25}$$

#### 4. Numerical solution

The similarity equations (22)–(24) with boundary conditions (25) are the set of coupled nonlinear boundary value problems. The Runge-Kutta fourth order technique along with shooting scheme is employed to this set of coupled nonlinear ordinary differential equations that transformed it into an initial value problem with unknown initial values. To determine these values we utilize few initial guesses. We found  $\eta_\infty = 6$  is a value appropriate for all the profiles which satisfies the boundary conditions (25) at infinity with step size of  $h = 0.001$  and the accuracy of error tolerance  $10^{-7}$  is used.

#### 5. Result and discussion

The reduced system of ordinary differential equations (22)–(24) along with the boundary conditions (25) is solved numerically by using the Runge-Kutta fourth order method along with shooting technique. The important features of unsteady MHD flow are outlined graphically through Figs. 2–8. Fig. 2 reveals the influence of the Grashof number  $Gr$  and the modified Grashof number  $Gm$  on the velocity profile. It is seen that the velocity changes remarkably when the Grashof and modified Grashof number increase, with the remaining present parameters which are in field of velocity treated as constant. It indicates the thermal buoyancy force effect on boundary layer to the force of viscous hydrodynamics. Hence the velocity accelerates due to enhancement in the thermal buoyancy forces and as the values of  $Gr$  and  $Gm$  escalate, it complements to the cooling of the plate.

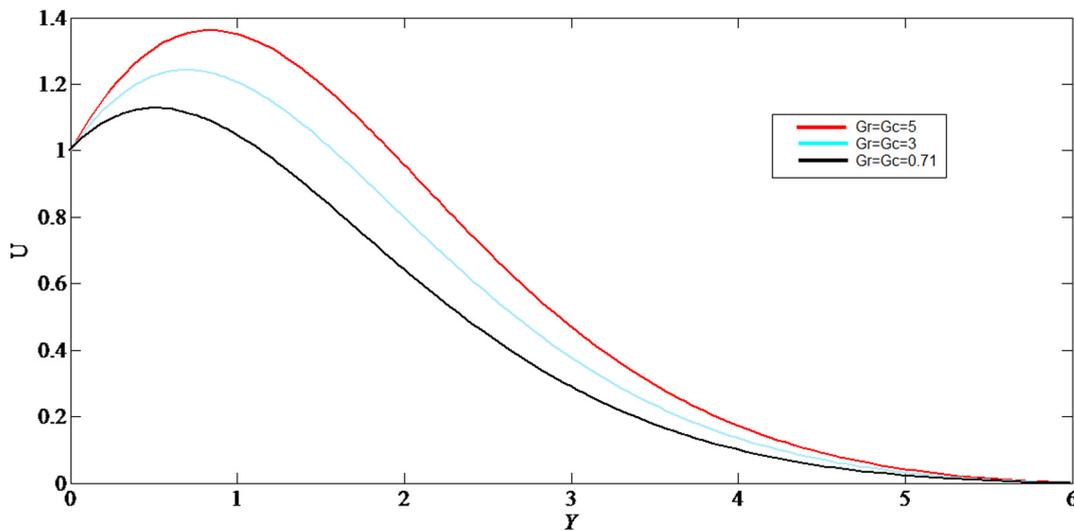


Fig. 2. Velocity profile for the values of  $Gr$  and  $Gm$ .

Fig. 3 is drawn to analyse the magnetic parameter effects on the velocity profile. As the strength of magnetic field increases, it decline the fluid velocity. As expected, the magnetic field maintains a retarding force over the flow of free convection.

Figs. 4 and 5 reflect the influence of the Prandtl number  $Pr$  on the velocity and the temperature profile. Fig. 4 demonstrates the increase in Prandtl number  $Pr$  causes a decrease in velocity. It is clear from Fig. 5 that the thickness of the thermal boundary layer decreases due to an addition in Prandtl number  $Pr$ . The smaller values of Prandtl number  $Pr$  increases the thermal conductivity, due to which the heat is rapidly diffused away compared to the higher values of  $Pr$  from the heated plate. Hence in present state, the rate of heat transfer decreases.

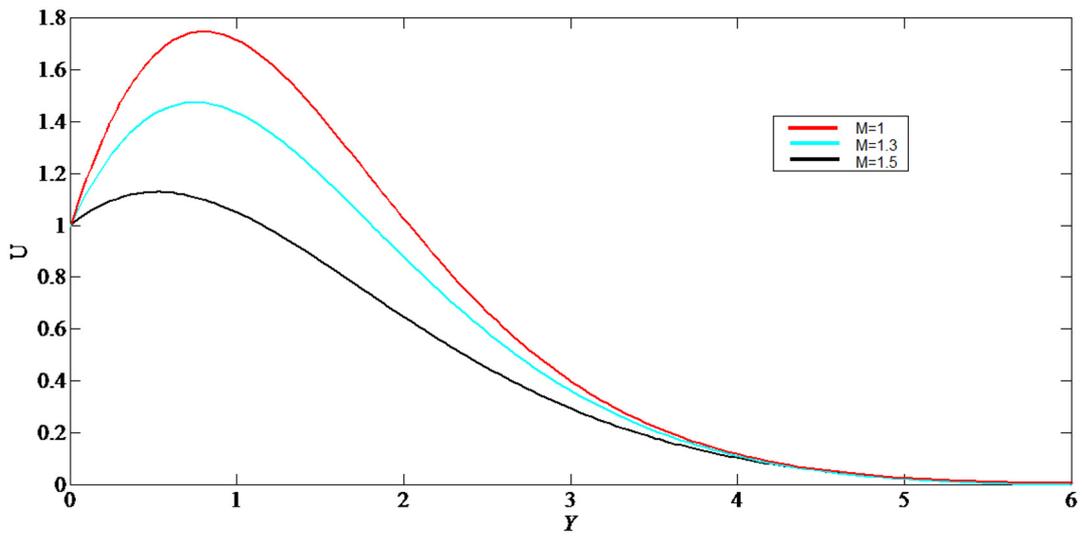


Fig. 3. Velocity profile for different values of magnetic parameter  $M$

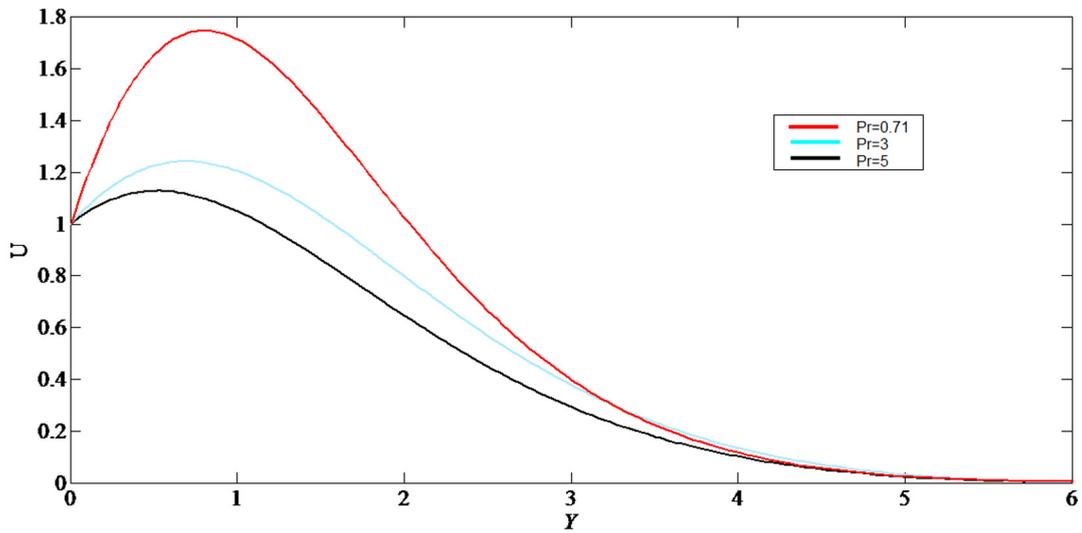


Fig. 4. Velocity profiles with distinct values of the Prandtl number  $Pr$

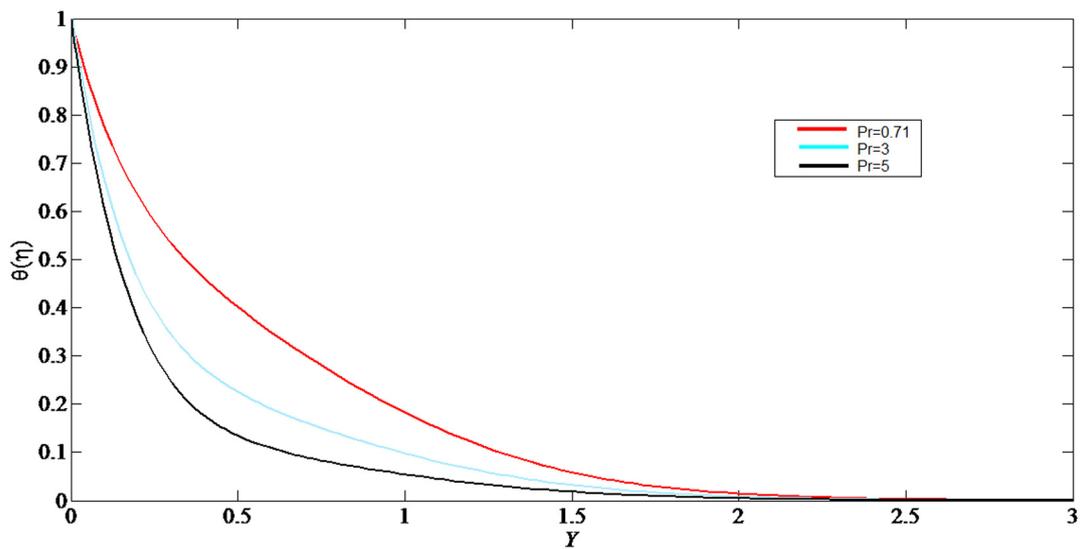


Fig. 5. Temperature profiles with distinct values of  $Pr$

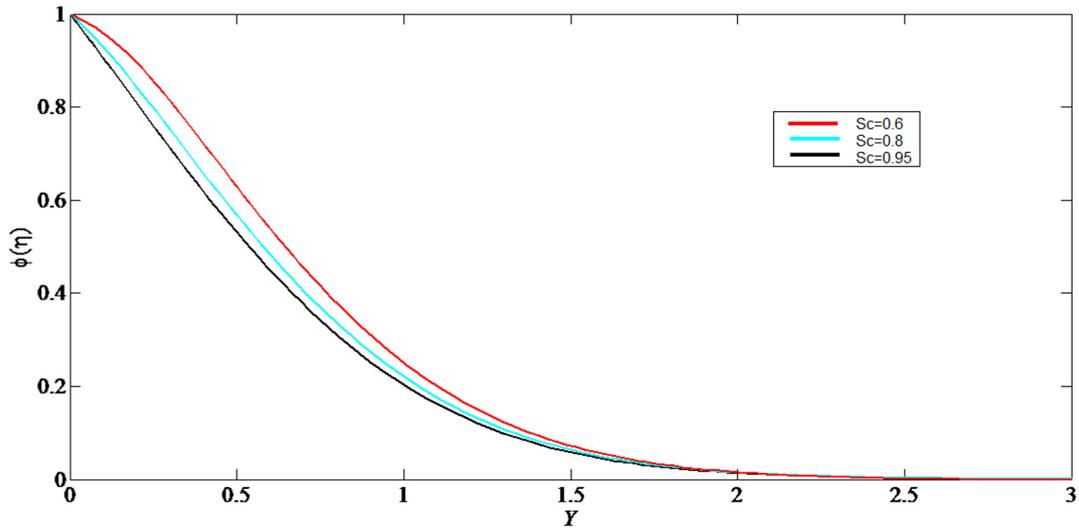


Fig. 6. Concentration profile of the distinct values of  $Sc$

Fig. 6 reveals the effect of  $Sc$  on the concentration profile. Increase in Schmidt number corresponds to fall down in the concentration profile. This affect on concentration buoyancy effects to drop off the velocity of the fluid. This contraction of concentration profile causes the reduction in the concentration boundary layer.

Next Fig. 7 reveals the consequences of radiation parameter  $R$  on the concentration profile. Here it is clearly found that the temperature profile falls down as the radiation rises up, which declines the thickness of the thermal boundary layer.

The response of the chemical reaction parameter  $Kr$  for different values over the velocity profile is shown in Fig. 8. Increase in  $Kr$  decreases the concentration boundary layer.

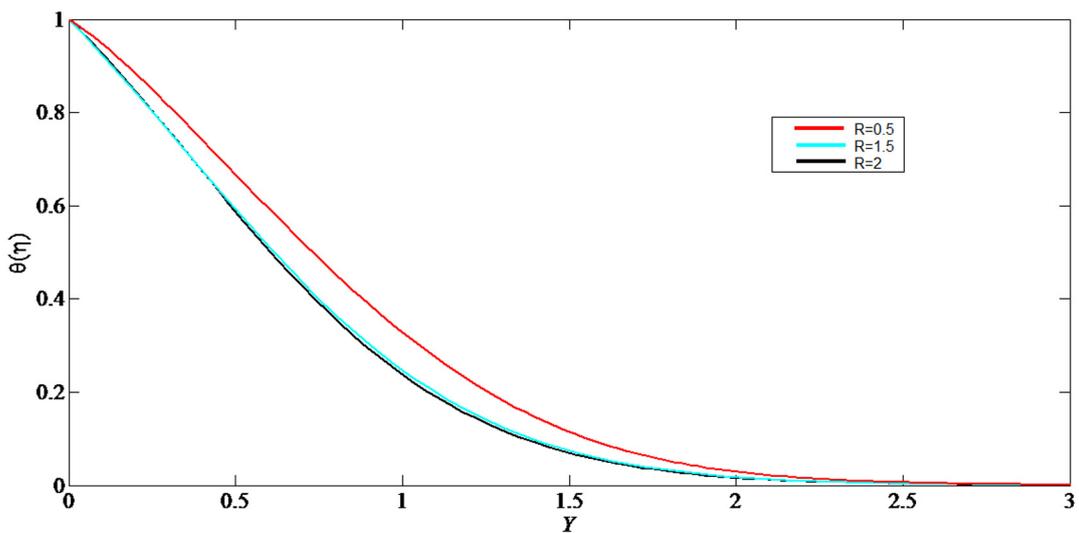


Fig. 7. Effects of  $R$  on the temperature profile

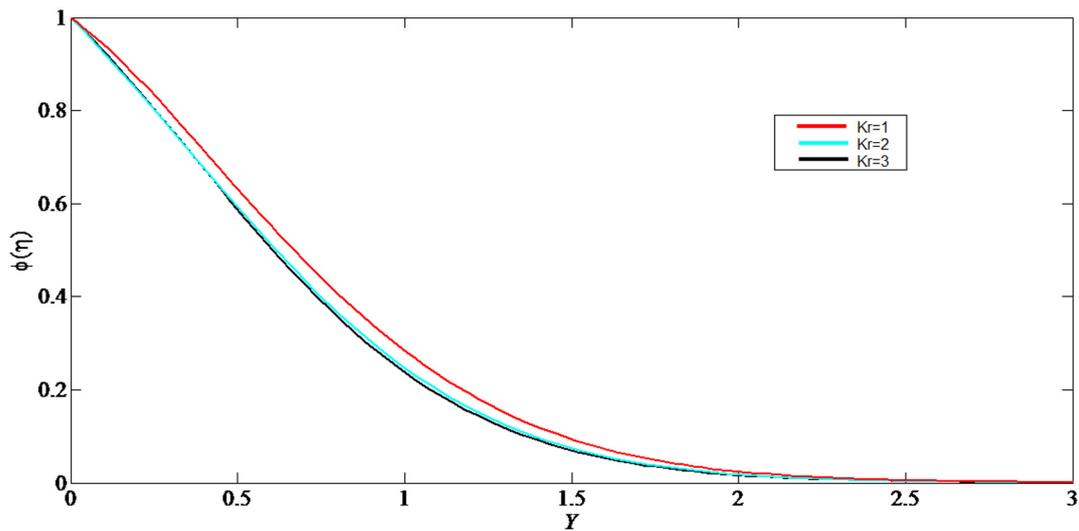


Fig. 8. Effects of  $Kr$  on the concentration profile

## 6. Conclusion

Similarity solutions are obtained for the unsteady mixed convection flow containing magnetic field considering radiation over vertical plate in porous medium by the Lie group symmetry analysis. The above study affirms the following deductions of physical interest on the velocity, temperature and the concentration profiles as well as the associated parameters of the flow field.

- The Grashof and modified Grashof numbers have accelerating effects on the velocity of the flow field.
- Velocity and temperature profile decreases due to increase in Prandtl number.
- The increase in Schmidt number declines the concentration profile.
- The increase in the radiation parameter leads to faster reduction in the temperature of flow field.

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