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Study of coupling between bending and torsional vibration of cracked rotor system supported by radial active magnetic bearings

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Abstract

The coupling of bending and torsional vibration due to the presence of transverse fatigue crack in a rotor system supported by radial active magnetic bearings (AMB) is investigated. For this purpose the modified stiffness matrix with six degrees of freedom per node is used and takes into account all the coupling phenomena that exists in a cracked rotor. The partial opening and closing of crack is considered by means of status of stress intensity factor along the crack edge. The equation of motion of rotor system is nonlinear due to response dependent non-linear breathing crack model and nonlinear force coupling introduced by AMB. A response of the rotor system is obtained by direct integration of nonlinear equation of motion. When the torsional harmonic excitation is applied to the rotor system with the crack then the sum and difference of torsional frequency around a bending natural frequency is observed in the lateral vibration spectrum. Influence of different values of crack parameters for two different speeds of rotor is investigated with help of frequency spectra.

Keywords: rotor system, active magnetic bearing, breathing crack

1. Introduction

Most of breakdowns in modern machinery are due to fatigue of the material. Transverse cracks in the rotor system can result in catastrophic damage and large economic loss as stated in the review of the literature [4] devoted to dynamics of rotor systems with cracks.

The transverse fatigue crack in a structural member introduces local flexibility, which for a beam element can be described by a local flexibility matrix. A full 6x6 local flexibility matrix for a transverse crack on a shaft was first introduced by Dimarogonas [2]. Computing of elements of this matrix is based on available expressions of the stress intensity factor and the associated expressions of the strain energy density function. In general, of the full 6x6 stiffness matrix available to model a crack, only the diagonal terms have been previously utilized for dynamic analysis of cracked rotors. In this work the full local flexibility matrix has been used to investigate coupling of various modes in a rotor system supported by radial AMB.

It is known, that during each revolution of the shaft the transverse crack can be opened, closed or it can periodically open and close [2]. The last case is referred to as so-called the breathing crack or the closing crack. When the rotor is operating at a steady-state speed far away from critical speed and without any transient excitation, the breathing of the crack can be approximated by an appropriate function of stiffness variation. A more realistic breathing crack model includes gradual opening and closing of the crack by means of the stress intensity factor along the crack edge. This model would be adaptable for all speed ranges and all

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types of excitation, steady-state as well as transient. Because of that the breathing model of crack has been adopted.

To the author's knowledge the dynamics of a cracked rotor system with an active feedback is investigated only in the articles [5], [7] and [8]. The model rotor system in these studies was consisting of massless flexible shaft with the transverse crack supported by two identical rigid bearings and one magnetic bearing and with disc located at the midspan. The active magnetic bearing and transverse crack are located in the proximity of disc and active magnetic bearing is incorporated into the computational model of rotor system by means of linear force coupling. The simple breathing model of crack developed in [4] is used.

2. Equation of motion of rotor system with transverse fatigue crack

It is assumed that the model rotor system with transverse fatigue crack has following properties: (i) the shaft is represented by a flexible beam-like body that is discretized into finite elements, (ii) shaft element is modelled on the basis of Bernouli-Euler beam theory with six degrees of freedom per node (i.e. two translations, two rotations, axial displacement and torsion), (iii) the stationary (i.e. non-rotating) part is considered to be absolutely rigid and fixed, (iv) the rotor is coupled with the stationary part through radial AMB and massless axial bearing, (v) the discs are considered to be absolutely rigid axisymmetric bodies, (vi) inertia and gyroscopic effects of the rotating parts are taken into account, (vii) material damping of the shaft and other kinds of damping are regarded as linear, (viii) the shaft contains a transverse fatigue crack, which influences only the stiffness matrix of the rotor system, (ix) the crack edge is assumed to be straight line, (x) the rotor is loaded by a gravitational force and periodic time histories forces and (xi) the rotor rotates at constant angular speed.

The equation of motion of the complete rotor system in the stationary co-ordinate system has the following form

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{q}}(t) + \left(\mathbf{B} + \eta_V \mathbf{K}_{SH} + \omega\mathbf{G}\right)\dot{\mathbf{q}}(t) + \left\{\mathbf{K}[\mathbf{q}(t)] + \omega\mathbf{K}_C\right\}\mathbf{q}(t) = \\ = \mathbf{f}_M(\mathbf{q}, \mathbf{i}) + \mathbf{f}_A(t) + \mathbf{f}_V, \end{aligned} \tag{1}$$

$$\mathbf{q}_{OP} = \mathbf{q}_{OP}(t), \qquad \mathbf{q}(0) = \mathbf{q}_0, \qquad \dot{\mathbf{q}}(0) = \dot{\mathbf{q}}_0, \tag{2}$$

where $\mathbf{M}, \mathbf{B}, \mathbf{G}, \mathbf{K}[\mathbf{q}(t)], \mathbf{K}_{SH}, \mathbf{K}_C$ are the mass matrix, the damping matrix (external damping and damping of material), the gyroscopic effects matrix, the stiffness matrix of the rotor system with the transverse crack, the stiffness matrix of the shaft and the circulation matrix of the rotor system respectively, $\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}$ are vectors of generalized nodal displacements, velocities and accelerations respectively, $\mathbf{f}_M, \mathbf{f}_A, \mathbf{f}_V, \mathbf{i}$ are vectors of magnetic forces, generalized forces exerting on the rotor system (external and constraint forces) and control currents passing by core of electromagnets respectively, while ω is the angular speed of a shaft, η_V is the coefficient of viscous damping of material and *t* is the time. Further details on element formulation and element matrices of a rotor system without a crack can be found elsewhere [6].

The investigated rotor system is supported by radial AMB consisting of four electromagnets, which are uniformly distributed around the bearing circumference. The motion of rotor in horizontal and vertical direction is controlled by two independent current PD feedback controllers. AMB are incorporated into the equation of motion (1) by means of nonlinear force coupling. Formulas for components of vector of magnetic force exerting on rotor in the horizontal and vertical direction and formulas of current PD feedback controller are given in [3].

3. Derivation of stiffness matrix of cracked shaft element

The finite elements employed are based on Bernouli-Euler beam theory. The influence of the crack is taken into account through modification of the stiffness matrix obtained by finite element formulation.

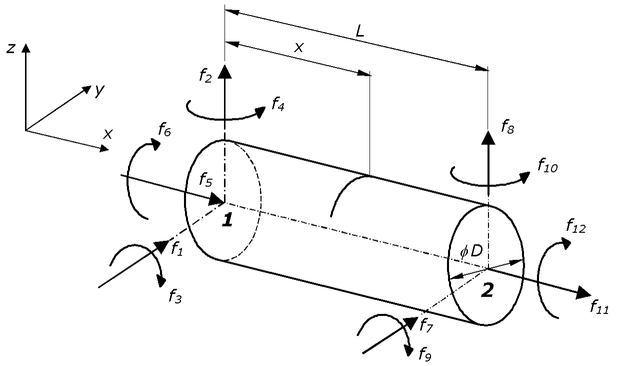

Fig. 1. A shaft element with the transverse crack.

Let us consider a shaft element with diameter D $(R=D/2)$, length L and a transverse crack situated at a distance x from the first node as shown in fig. 1. The shaft element is loaded by shear forces f_1, f_2 and f_7, f_8, bending moments f_3, f_4 and f_9, f_{10}, axial forces f_5 and f_{11} and torsional moments f_6 and f_{12}.

The presence of the transverse fatigue crack in the shaft element introduces a local flexibility, which is described by a local flexibility matrix $\mathbf{C}^{\mathrm{e}}_{\mathrm{cr}}$. The dimension of the local flexibility matrix is in general dependent on the number of degrees of freedom of the shaft element. Thus, in this case the local flexibility matrix is six by six matrix.

Calculation of the flexibility matrix of cracked shaft element $\mathbf{C}^{\mathrm{e}}$ is determined as a sum of local flexibility matrix of cracked shaft element $\mathbf{C}^{\mathrm{e}}_{\mathrm{cr}}$ and flexibility matrix of shaft element without the crack $\mathbf{C}^{\mathrm{e}}_{\mathrm{ncr}}$ (see [2]).

Using concepts of fracture mechanics the elements of local flexibility matrix due to crack are given according to [2] by the following expression

$$\left(c_{\mathrm{i,j}}\right)_{\mathrm{cr}} = \frac{\partial^2}{\partial f_{\mathrm{i}}\, \partial f_{\mathrm{j}}} \int_0^a \int_{-s_1}^{s_2} J(\alpha)\, d\alpha\, d\gamma \,, \tag{3}$$

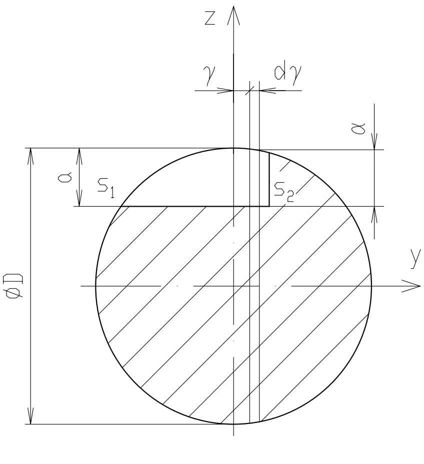

Fig. 2. The cross-section of shaft with the crack.

where f_{i}, f_{j} are corresponding loads (forces and moments) for i, j = 1, 2, ..., 6$^{\mathrm{th}}$ load index, α is the immediate depth of the crack, a is the total depth of the crack, s_1 is half a width of crack, s_2 is width of crack, γ is the distance between rectangular strip of width $d\gamma$ and vertical axis of the shaft (see fig. 2) and $J(\alpha)$ is the strain energy density function.

The increase in the displacement due to the presence of the crack is derived for the first node of shaft element. The complete stiffness matrix $\mathbf{K}^{\mathrm{e}}_{\mathrm{cr}}$ is then obtained by means of transformation matrix $\mathbf{T}$, which is obtained by considering the static equilibrium of the finite element [1]. The stiffness matrix of cracked element in rotor-fixed coordinate system has the following form

$$\mathbf{K}^{\mathrm{e}}_{\mathrm{cr}} = \mathbf{T}\left(\mathbf{C}^{\mathrm{e}}\right)^{-1}\mathbf{T}^{\mathrm{T}} . \tag{4}$$

The stiffness matrix of cracked element (4) has to be transferred into the stationary coordinate system and then it is placed into correct position in the global stiffness matrix of the cracked rotor system.

4. Incorporation of breathing of crack into computational model of rotor system

Size of elements of local flexibility matrix (3) depends on size of crack opening or closing (fig. 2). For fully open crack the integration limits in expression (3) are from $-s_1$ to s_1. The breathing of crack is incorporated into the computational model of the rotor system by using concept of crack closure line (CCL), which was proposed in the article [1]. The CCL is an imaginary line perpendicular to the crack edge, which is separating opened and closed part of crack (see fig. 2). It is assumed that crack edge is straight line and its total length is equal to $2s_1$. The crack edge is divided into lines of equal length and stress intensity factor is calculated in the middle of each line. Positive stress intensity factor indicates tensile stress field and the crack is open. Negative stress intensity factor indicates compressive stress field and crack is closed at that point. The total stress intensity factor is given by

$$K = K_{13} + K_{14} + K_{I5}, \tag{5}$$

$$K_{13} = \sigma_3\sqrt{\pi\alpha}\; F_2(\alpha/h),\; K_{14} = \sigma_4\sqrt{\pi\alpha}\; F_1(\alpha/h),\; K_{15} = \sigma_5\sqrt{\pi\alpha}\; F_1(\alpha/h), \tag{6}$$

$$\sigma_3 = \frac{2h(f_3 + f_2 x)}{\pi R^4},\; \sigma_4 = \frac{4\gamma(f_4 - f_1 x)}{\pi R^4},\; \sigma_5 = \frac{f_5}{\pi R^2},\; h = 2\sqrt{R^2 - \gamma^2}, \tag{7}$$

where F_1 and F_2 are configuration correction factors for stress intensity factors and meaning of other symbols is evident from fig. 1 and fig. 2.

5. Computation of response of rotor system with breathing crack on excitation by centrifugal forces due to unbalance of rotating parts

The appropriate direct integration method has been employed to obtain transient response of the rotor system with breathing crack on excitation by unbalance of the rotating parts. From the transient response the steady-state response of a rotor system is estimated. The nonlinear equation of motion (1) of examined a rotor system are solved in the stationary co-ordinate system. Due to breathing of crack the stiffness matrix of the cracked rotor system $\mathbf{K}[\mathbf{q}(t)]$ is dependent on calculated response of the rotor system. Stiffness matrix of the rotor system with the transverse crack $\mathbf{K}[\mathbf{q}(t)]$ is assumed to be constant for short period of time (e.g. one degree of rotation), because variation of integration limits in the expression (3) is insignificant. After every short period of time the stiffness matrix needs to be updated to take into account the breathing behaviour of the cracked rotor. For this purpose nodal forces in the stationary co-ordinate system $\mathbf{f}_S$ are calculated

$$\mathbf{f}_S = \{\mathbf{K}[\mathbf{q}(t)] + \omega\mathbf{K}_C\}\mathbf{q}(t), \tag{8}$$

where $\mathbf{q}$ is the new response vector obtained by integration of equation of motion (1) and $\mathbf{K}[\mathbf{q}(t)]$ is the stiffness matrix of the rotor system with the transverse crack in the stationary co-ordinates. Nodal forces calculated in the stationary co-ordinate system $\mathbf{f}_S$ must be transformed into the rotor-fixed-co-ordinates $\mathbf{f}_R$ by an appropriate transformation matrix $\mathbf{T}_R$ (see [1]). By means of these nodal forces $\mathbf{f}_R$ the total stress intensity factor (5) along the crack edge is calculated. Therefore the appropriate integration limits in the expression (3) for computing corresponding stiffness matrix of a cracked element are known.

6. Test rotor system with transverse fatigue crack

Presented computational procedures were built up in the computer system MATLAB (R2007a) and tested by means of computer simulations on the test rotor system (fig. 3).

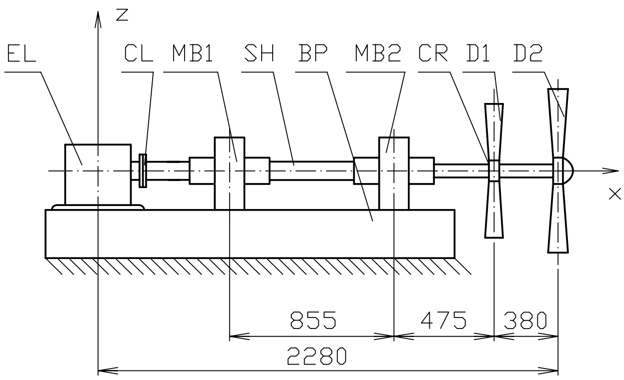

Fig. 3. A sketch of the test rotor system.

The investigated rotor system is consisting of the shaft (SH) driven by electromotor (EL) through the clutch (CL) and two discs (D1, D2) fixed on cantilever as shown in the fig. 3. The rotor is connected with the bed plate (BP) by means of two radial AMB (MB1 and MB2). The transverse fatigue crack (CR) is located on the shaft (SH) in the proximity of disc (D1) on the side of magnetic bearing (MB2).

The shaft in the computational model is represented by a beam-like body that is discretized into twenty four finite elements of equal length (95 mm) and more detailed data are given in [3]. Rayleigh damping coefficients α, β and the coefficient of viscous material damping η_V are assumed equal to 4 s^{-1}, 0 s and 2.0 10^{-6} s^{-1} respectively. The current PD feedback controllers and MB1 and MB2 are assumed to be identical and their parameters are given in [3]. The discrete axial bearing stiffness is in the axial direction $k_x = 1 \cdot 10^5\ \mathrm{N\,m^{-1}}$ and torsional stiffness is $k_t = 1\,\mathrm{N\,m\,rad^{-1}}$. The eccentricity of both discs is equal to 0.005 mm and is located on positive axis z (see fig. 2).

The numerical simulations were performed with several depths of crack a = 13.2 mm, a = 26.4 mm, a = 39.6 mm which are corresponding to 15%, 30%, 45% of diameter of shaft.

In some cases of computation of response the additional torsional excitation is applied. The time history of torsional moment is $M_t = M_A \sin(\omega_t\, t)$, where $M_A = 300\,\mathrm{N\,m}$ is the amplitude of torsional moment and ω_t is the torsional excitation frequency.

7. Results of numerical experiments with test rotor system

The assembly of the Campbell's diagram of uncracked rotor system is made in the neighbourhood of equilibrium position. The results of several values of critical speeds and its higher harmonics are stated in tab. 1.

Modes of vibration and critical speeds $[\mathrm{rad\,s^{-1}}]$	
1^{st} axial mode	18.1
1^{st} bending mode	76.1 110.5
2^{nd} bending mode	399.0 564.1
1^{st} torsional mode	612.1
3^{rd} bending mode	716.3 929.7

Tab. 1. Critical speeds.

The response of a rotor system on excitation by centrifugal forces due to unbalance of the rotating parts has been solved with direct integration method. In the computer system MATLAB the implemented functions "ode15s" or "ode23" were used. After dying out of the initial transient component from obtained response the steady-state response is estimated. The presented frequency spectra were computed from steady-state response by applying discrete Fourier transform.

Since cracked shaft is causing coupling of longitudinal, torsional and lateral bending vibration the response of uncracked rotor system is computed with and without torsional excitation. Angular speed is 15.2 $\mathrm{rad\,s^{-1}}$, which is approximately 1/5th of first bending critical speed (e.g. 76.1 $\mathrm{rad\,s^{-1}}$). Then the harmonic torsional excitation with $\omega_t = 76.1\,\mathrm{rad\,s^{-1}}$ was applied at the sixth node of discretized rotor system (at the disc location). The computed responses shown in fig. 4 and fig. 5 were equivalent to responses computed without torsional excitation. In the case of uncracked rotor system the coupling mechanism between torsional excitation and the response in lateral direction doesn't exist. Response in the torsional direction shows the presence of torsional excitation frequency (fig. 6). The response of rotor system in vertical direction is shown in left fig. 5 and the corresponding frequency spectra contains except rotational frequency its second harmonic frequency with much smaller amplitude caused by nonlinear magnetic forces (right fig. 5).

The response of rotor system with the crack of depth 13.2 mm and angular speed 15.2 $\mathrm{rad\,s^{-1}}$ excited only by centrifugal forces due to unbalance of the rotating parts is now investigated. The lateral bending vibration in both horizontal (fig. 7) and vertical (fig. 8) direction contain rotational frequency and its higher harmonics. Also in the longitudinal (fig. 9) and torsional (fig. 10) vibration the rotational frequency and its higher harmonics were identified. Higher harmonics indicate the coupling phenomenon between the bending and the longitudinal vibration as well as between the bending and the torsional vibration. The unbalance excitation applied on rotor system with cracked shaft generated lateral, axial and torsional vibration.

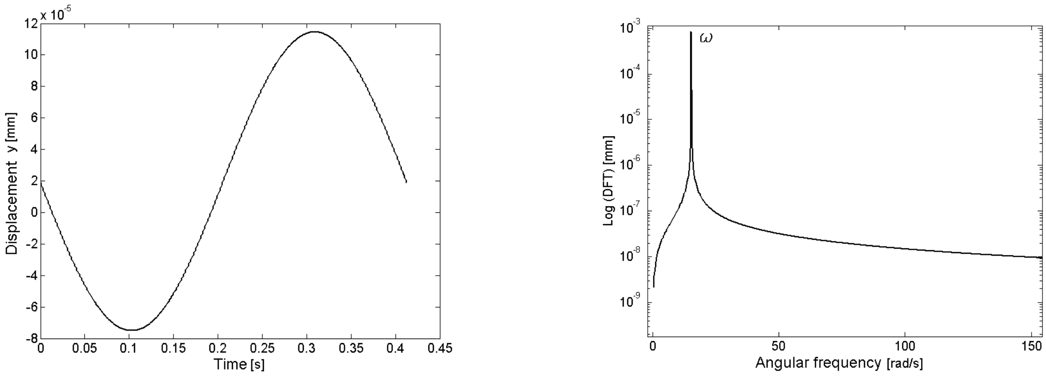

Fig. 4. Time domain response in horizontal direction of an uncracked rotor system with torsional excitation (left) and corresponding frequency domain response (right).

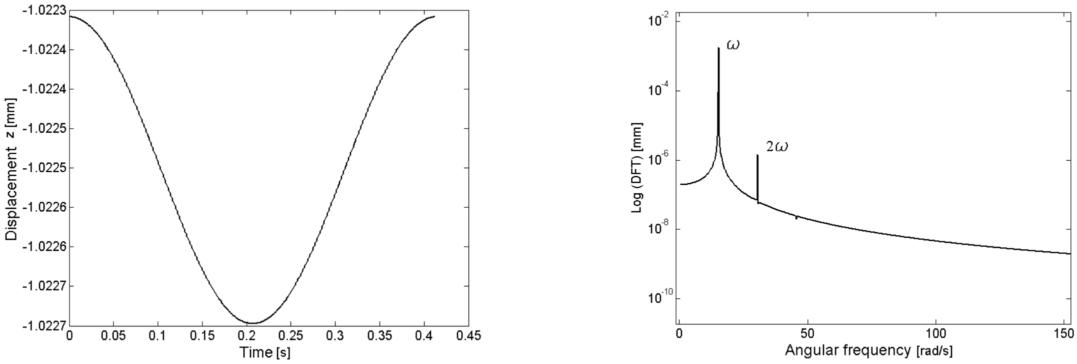

Fig. 5. Time domain response in vertical direction of an uncracked rotor system with torsional excitation (left) and corresponding frequency domain response (right).

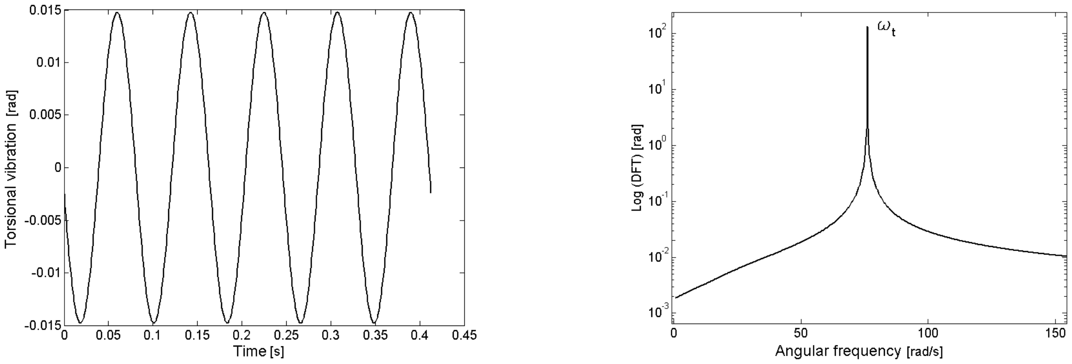

Fig. 6. Torsional response in time domain of an uncracked rotor system with torsional excitation (left) and corresponding frequency domain response (right).

Next analysis has been carried out with parameters of rotor system from previous analysis and in addition the torsional excitation with frequency $\omega_t = 76.1\ \mathrm{rad\,s^{-1}}$ was applied, which is equal to the 1st bending critical speed. Although in this case the torsional excitation is applied on the rotor system, the peak of the 1st bending critical speed (left fig. 11) has the similar value as for rotor without torsional excitation (right fig. 7). The amplitude variations of frequency components of response in the horizontal direction for three depths of crack are shown in right fig. 11. All frequency components are in the same way sensitive on depth of crack.

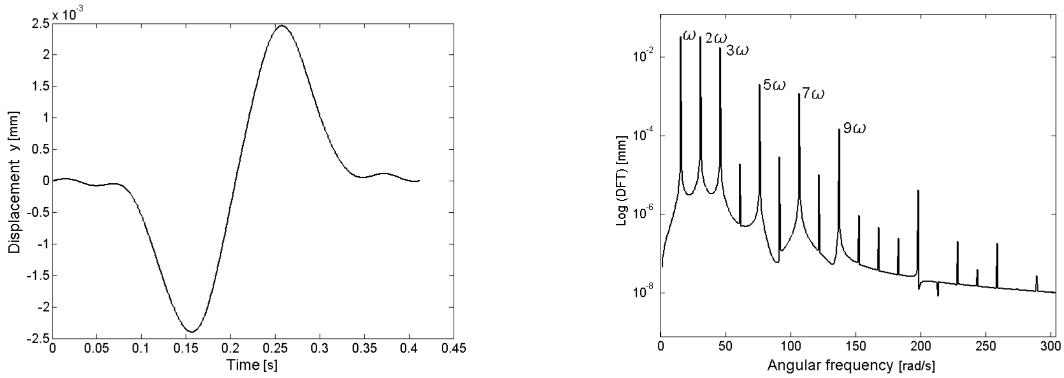

Fig. 7. Time domain response in horizontal direction of a cracked rotor system (left) and corresponding frequency domain response (right).

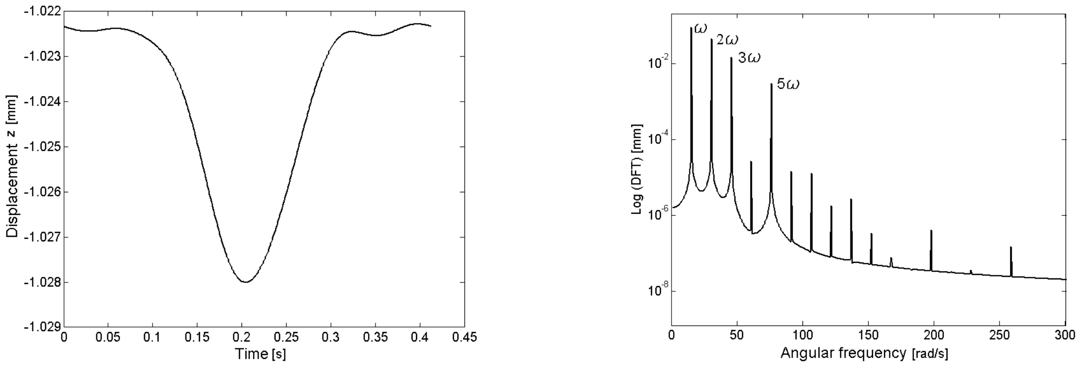

Fig. 8. Time domain response in vertical direction of a cracked rotor system (left) and corresponding frequency domain response (right).

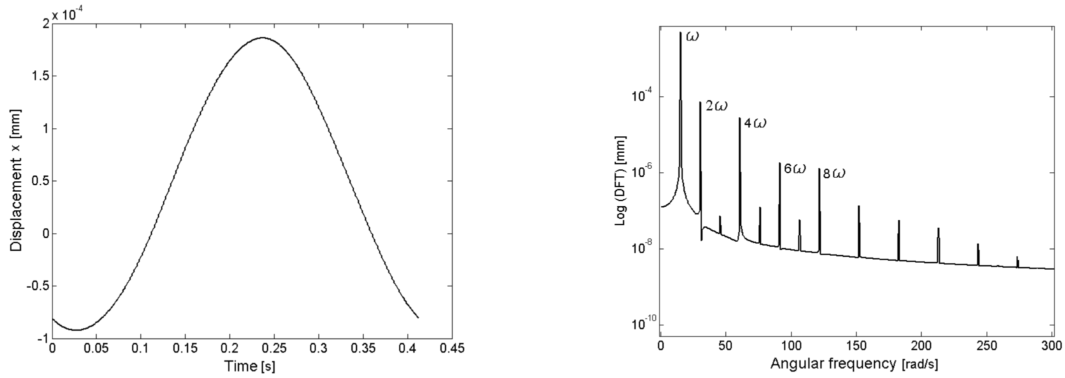

Fig. 9. Time domain response in axial direction of a cracked rotor system (left) and corresponding frequency domain response (right).

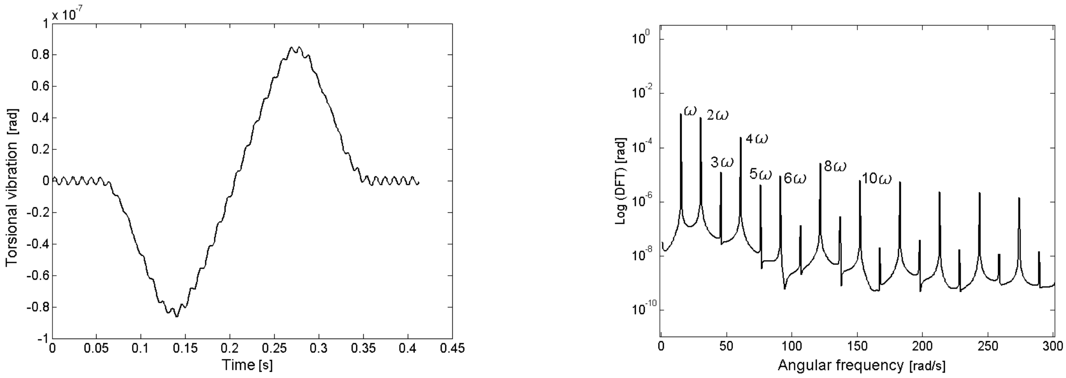

Fig. 10. Time domain torsional response of a cracked rotor system (left) and corresponding frequency domain response (right).

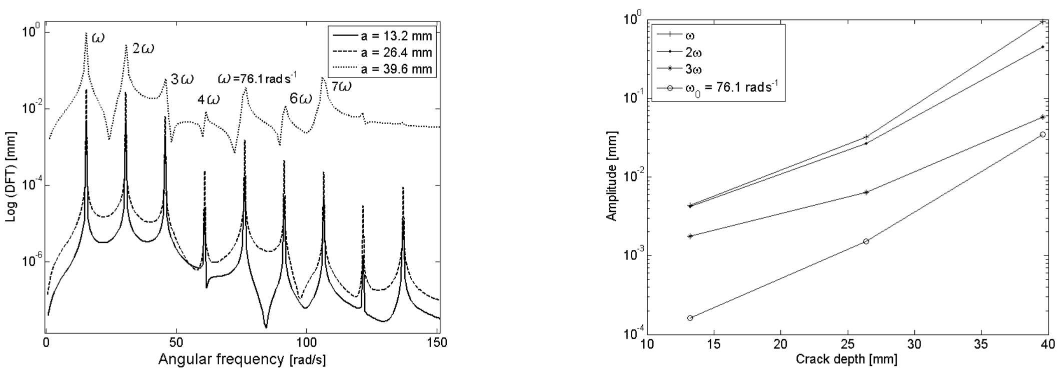

Fig. 11. Frequency domain response in horizontal direction of a cracked rotor system (left) and variation of amplitudes of rotational frequency and its higher harmonics (right).

The cracked rotor with depth of crack a = 13.2 mm rotating at angular speed 17.8 $\mathrm{rad\,s^{-1}}$, which is not equal to integer fraction of bending critical speed, excited by torsional moment with frequency equal $\omega_t = 43.6\ \mathrm{rad\,s^{-1}}$ (fig. 15) is now studied. The spectra of lateral vibration (right fig. 12 and right fig. 13) and axial vibration (right fig. 14) shows the rotational frequency and its higher harmonics. Then lateral vibration spectra (right fig. 12 and right fig. 13) contain around the torsional excitation frequency ω_t sums and differences of rotational frequency and torsional excitation frequency like are $\omega_t \pm \omega$ and $\omega_t \pm 2\omega$. Arguably some of higher harmonics of rotational frequency in these frequency spectra are caused by interaction between nonlinearities of magnetic forces and nonlinear model of breathing crack. The side frequencies are present due to interaction between torsional excitation frequency and the lateral unbalance excitation.

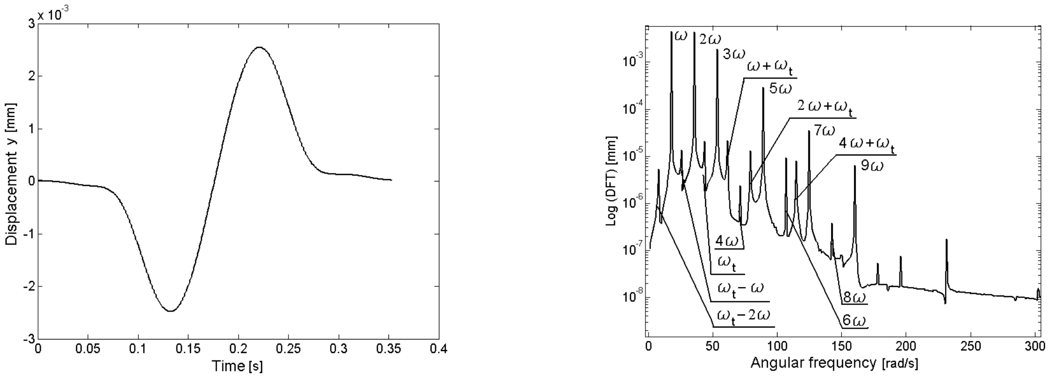

Fig. 12. Time domain response in horizontal direction of a cracked rotor system with torsional excitation (left) and corresponding frequency domain response (right).

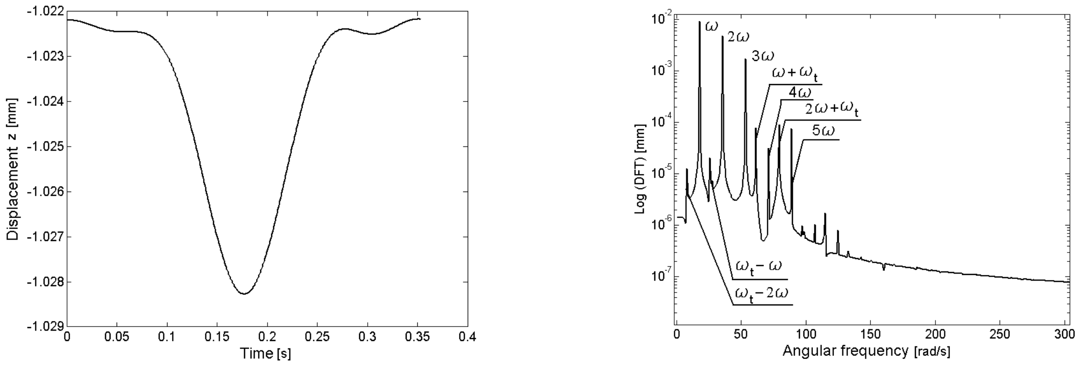

Fig. 13. Time domain response in vertical direction of a cracked rotor system with torsional excitation (left) and corresponding frequency domain response (right).

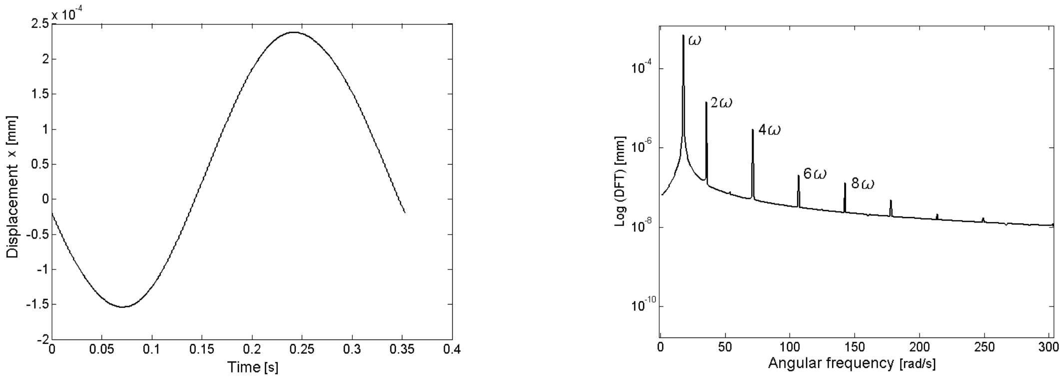

Fig. 14. Time domain response in axial direction of a cracked rotor system with torsional excitation (left) and corresponding frequency domain response (right).

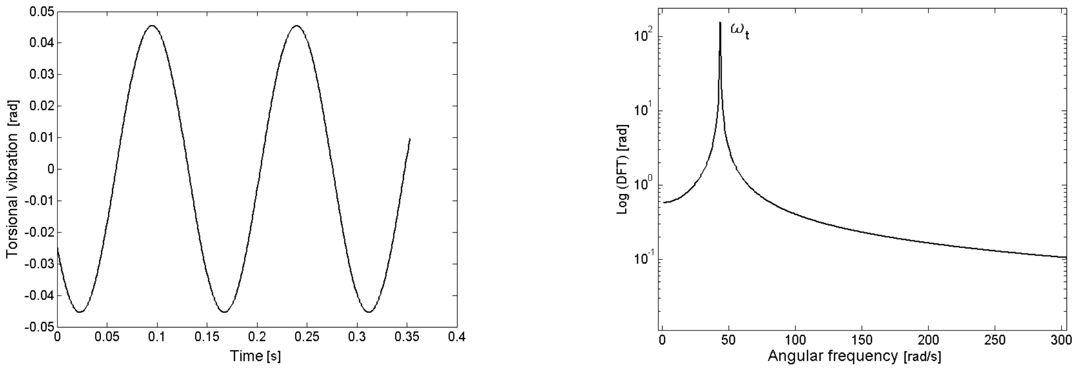

Fig. 15. Time domain torsional response of a cracked rotor system with torsional excitation (left) and corresponding frequency domain response (right).

8. Conclusion

Finite element method has been used for study of steady-state response of rotor system supported by radial AMB with the transverse breathing crack in a shaft. The steady-state response of a rotor system on excitation by centrifugal forces due to unbalance of the rotating parts and in some special cases additional torsional harmonic excitation is studied in both time and frequency domain. The unbalance excitation applied on the cracked rotor system in lateral direction excited response in torsional and axial direction, which is indicating coupling mechanism between bending and torsional vibration and between bending and longitudinal vibration. When an angular speed of rotor was not an integer fraction of its bending critical speed, the frequency spectra contained side frequencies around the frequency of torsional excitation. The results presented in this work can therefore help with detection of cracks in the shaft.

Acknowledgement

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