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Similarity solution of the shock wave propagation in water

M. Müller[a,*]

[a] *Department of Power Engineering Equipment,Technical University of Liberec, 46117 Liberec, Czech Republic*

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Abstract

This paper presents the possibility of calculation of propagation of a shock wave generated during the bubble collapse in water including the dissipation effect. The used semi-empirical model is based on an assumption of similarity between the shock pressure time profiles in different shock wave positions. This assumption leads to a system of two ordinary differential equations for pressure jump and energy at the shock front. The NIST data are used for the compilation of the equation of state, which is applied to the calculation of the shock wave energy dissipation. The initial conditions for the system of equations are obtained from the modified method of characteristics in the combination with the differential equations of cavitation bubble dynamics, which considers viscous compressible liquid with the influence of surface tension. The initial energy of the shock wave is estimated from the energy between the energies of the bubble growth to the first and second maximum bubble radii.
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1. Introduction

Cavitation is phenomenon which occurs in different kinds of systems working with liquids. The cavitation damage can be caused by several effects, but the most important are the liquid jet and shock wave produced during the bubble collapse. The direct experimental detection of the shock wave is complicate because of its short duration. The measurement has to be supported by an appropriate numerical method in these situations. The basic method used for the calculation of the shock wave propagation is the method of characteristics, which does not include dissipation of energy. This results in that the energy of the shock wave is constant and only the peak pressure and shock wave velocity decrease with increasing radial position of the shock wave. This is caused by using isothermal equation of state, which is simple in use, but it disables any access to the energy dissipation, which usually causes temperature increase after passing the shock wave.

The semi-empirical solution, which is presented here, was introduced by Brinkley and Kirkwood in [1]. It simplifies the solution of system of partial differential equations describing the shock wave propagation to set of two ordinary differential equations for the peak pressure at the shock wave and the shock wave energy as a function of radial coordinate. The energy of the shock wave is usually determined from the pressure-time profile in one position of the shock wave, but it can also be obtained from the difference of the energies needed for the bubble growth to the first and to the second maximum radii. However, the position where the shock wave has developed has to be determined using the method of characteristics in the combination with the equation of bubble dynamics as it is presented in [4].

[*]Corresponding author. Tel.: +420 485353428, e-mail: milos.muller@tul.cz.

2. The equation of state and dissipated energy

2.1. Tait´s equation of state

The most common equation of state (EOS) for liquids is Tait´s equation, which represents the dependence between the liquid density and pressure. In the presented case, it is convenient to use the EOS in isoentropic form as (see e.g. [5])

$$\frac{\rho(p,s)}{\rho(p=0,s)} = \left(\frac{B(s)+p}{B(s)+p_0}\right)^{\frac{1}{n}}, \tag{1}$$

where for water $n=7$. The coefficient $B(s)$ can be expressed as a function of the sound velocity c_0 and the density ρ_0 at pressure p_0 as

$$B(s) = \frac{\rho_0 c_0}{n}. \tag{2}$$

$$c_0(T) = k_{4c}T^4 + k_{3c}T^3 + k_{2c}T^2 + k_{1c}T + k_{0c} \tag{3}$$

$$\rho_0(T) = k_{4\rho}T^4 + k_{3\rho}T^3 + k_{2\rho}T^2 + k_{1\rho}T + k_{0\rho}. \tag{4}$$

For the calculation of the dissipated energy the heat capacity at the pressure p_0 as function of temperature is needed as

$$c_p(T) = k_{4cp}T^4 + k_{3cp}T^3 + k_{2cp}T^2 + k_{1cp}T + k_{0cp}. \tag{5}$$

The material relations used in this work were obtained by fitting data from NIST web database [3]. The coefficients for the material relations are given in the Tab. 1.

	k_0	k_1	k_2	k_3	k_4
c_0	-1.303670E+04	1.469587E+02	-5.629782E-01	9.801810E-04	-6.599914E-07
c_p	4.040970E+04	-4.307597E+02	1.921642E+00	-3.815560E-03	2.828450E-06
ρ_0	1.120944E+03	2.378523E+01	-9.867555E-02	1.817291E-04	-1.288819E-07

Tab. 1. Coefficients for Eqs. (3), (4) and (5).

2.2. Evaluation of dissipated enthalpy at the shock wave

If the shock wave passes through a position in liquid, which had temperature T_0 and pressure p_0, the temperature and pressure increase up to the values p and T as it can be seen in Fig. 1. The specific enthalpy increment experienced by the fluid ΔH can be obtained from Rankine-Hugoniot conditions at the shock wave as (see e.g. [2])

$$\Delta H = \frac{p}{2}\left(\frac{1}{\rho} + \frac{1}{\rho_0}\right), \tag{6}$$

where ρ is liquid density at pressure p and ρ_0 is density at pressure p_0. Having passed the shock wave the pressure in the liquid reaches again the pressure p_0 along an adiabatic curve

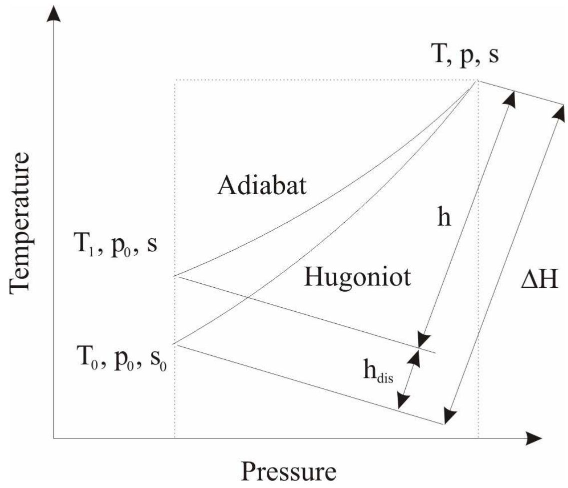

Fig. 1. Distribution of the total energy of the shock wave.

but the temperature returns to a higher value T_1 due to the dissipation process. The estimation of the temperature T_1 is the key for obtaining the dissipated energy. Based on the consideration, the enthalpy increment ΔH can be evaluated as a sum of undissipated and dissipated enthalpy as

$$\Delta H = h + h_{dis} . \tag{7}$$

The undissipated enthalpy h can be expressed using the equation of state (1) according to the Fig. 1. as

$$h = \frac{2c_1^2}{n-1}\left(\left(\frac{\rho}{\rho_1}\right)^{\frac{1}{n}} - 1\right) \tag{8}$$

and dissipated enthalpy h_{dis} can be evaluated from the specific heat data, which is known as an explicit function of temperatures T_0 and T_1 as

$$h_{dis} = \int_{T_0}^{T_1} c_p(T_0, T_1)\, dT . \tag{9}$$

Having substituted Eqs. (6) and (8) into Eq. (7) one obtain

$$\frac{h_{dis}}{c_1^{\,2}} = -\frac{2}{n-1}\left[\left(\frac{\rho}{\rho_1}\right)^{n-1} - 1\right] + \frac{p}{2\rho_1}\left(\frac{\rho_1}{\rho} + \frac{\rho_1}{\rho_0}\right). \tag{10}$$

The dissipated enthalpy h_{dis} is eliminated from the Eq. (10) using Eq. (9), where the heat capacity is obtained from Eq. (5). Finally, the ratio ρ_1/ρ can be eliminated from Eq. (10) using Eqs. (1) and (2) as

$$p = \frac{c_1^2 \rho_1}{n}\left[\left(\frac{\rho}{\rho_1}\right)^{n-1} - 1\right]. \tag{11}$$

For given pressure p, the temperature T_1 can be obtained from the Eq. (4) which is then used in the Eq. (6) for the calculation of the dissipated enthalpy h_{dis}. Note that the quantities evaluated at temperature T_1 are marked with subscript "1". Some results for the dissipated enthalpy h_{dis} and temperature increase T_1 compared with previously published data [5] are given in Tab. 2.

T_0=293K	NIST			Richardson[5]
p [MPa]	ΔT1 [K]	T1 [K]	h_{dis} [J/kg]	h_{dis} [J/kg]
0	0.0	293.0	0.0	0.0
250	0.2	293.2	1046.0	x
500	1.2	294.2	5152.0	5570.0
1000	5.4	298.4	22480.0	23450.0
500	11.5	304.5	47978.0	49350.0
2000	18.8	311.8	78484.0	80050.0
2500	26.9	319.9	112547.0	115000.0

Tab. 2. Dissipated enthalpy and temperature increase calculated as function of the pressure at the shock wave.

3. Semi-empirical model - Similarity solution

The model used in this work for the calculation of the shock wave propagation has been developed by Brinkley and Kirkwood and is available in works [1] and [2]. As the aim of this work is not reformulation of the theory, but the new definition of the dissipated enthalpy (9), which is included in the model, only the main idea of the derivation and the final relations will be given here. The aim of the solution is to find out how the pressure peak *p* varies with the shock wave position *r*.

Propagation of spherical shock wave can be completely described by Euler's equations in spherical coordinates. These two equations together with the Rankine-Hugoniot conditions at the shock front give set of three equations for four unknown partial derivatives of pressure and particle velocity ∂p/∂t, ∂p/∂r, ∂v/∂t and ∂v/∂r. If this set of equations is supplemented by one additional equation, it is possible to solve for each derivative and formulate two ordinary differential equations for the shock wave energy and shock pressure as functions of radial co-ordinate *r*. This additional relation introduces into the model an empirical shape of

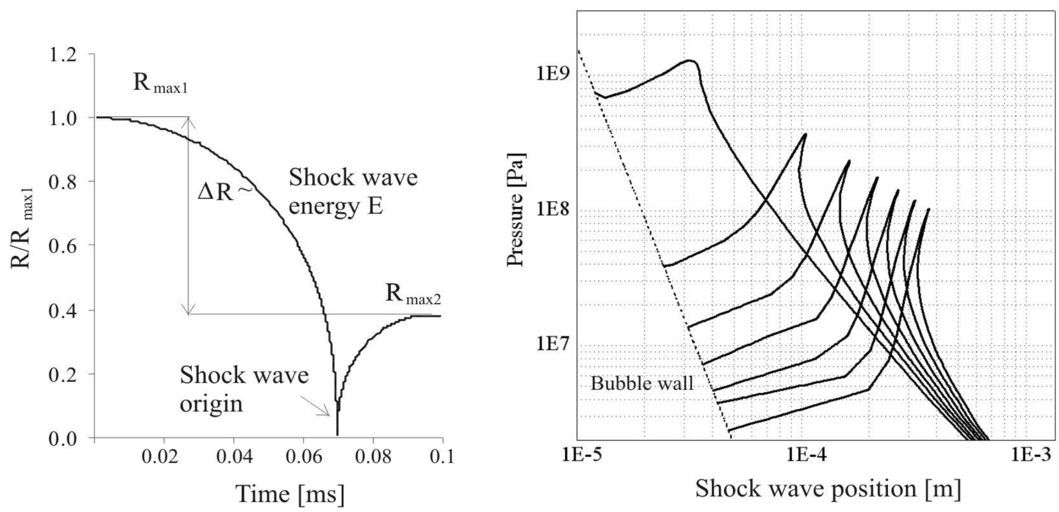

Fig. 2. Bubble collapse (left) and the shock wave propagation (right) in water.

the pressure time profile, which is similar at each shock wave position. The resulting equations of the model are

$$\frac{dp}{dr}=\frac{r^2p^3}{E}\frac{1}{\rho_0u^2}\frac{1-\dfrac{\rho_0u}{\rho c}}{2(1+g)-\left(1-\dfrac{\rho_0u}{\rho c}\right)}-\frac{p}{r}\frac{\left(\dfrac{4\rho}{\rho_0}+2\left(1-\dfrac{\rho}{\rho_0}\right)\left(1-\dfrac{\rho_0u}{\rho c}\right)\right)}{2(1+g)-\left(1-\dfrac{\rho_0u}{\rho c}\right)}. \tag{12}$$

$$\frac{dE}{dr}=-\rho_0r^2h_{dis}, \tag{13}$$

$$g=1-\frac{p}{u}\frac{\partial u}{\partial p},\ u=\frac{p}{\rho}\left(p\left(\frac{1}{\rho_0}-\frac{1}{\rho}\right)\right)^{-0.5},\ c=c_1\left(\frac{\rho}{\rho_1}\right)^{\frac{n-1}{2}}, \tag{14}$$

where E is the shock wave energy, u is the shock wave velocity and c is speed of sound. For detail description of the model, the original report [1] has to be consulted.

4. Calculation using the semi-empirical model

The model will be tested on compression of spherical bubble filled with ideal gas of pressure 2000 Pa from initial radius R_{1max} = 0.75 mm and following expansion of the bubble on radius R_{2max} = 0.22 mm as it is given in Fig. 2. The system of ordinary differential equations (12) and (13) require two boundary conditions. These conditions are the shock wave energy E and shock pressure p at given position. The initial energy of the shock wave can be obtained from the difference between the energies needed for the bubble growth to the first and to the second maximum radii as (see e.g. [2])

$$E=\frac{4}{3}\pi p_\infty\left[R_{1\max}^3-R_{2\max}^3\right]. \tag{15}$$

The pressure of the shock wave at given position has to be calculated using method of characteristics, which is described in [4]. For the given bubble radii, the results obtained from the model presented in [4] are given in Fig. 2. The boundary conditions for the solution of Eq. (12) and (13) are r = 4e-4 mm, E = 1.5e-4 J and p = 1.1e9 Pa. The results of the solution

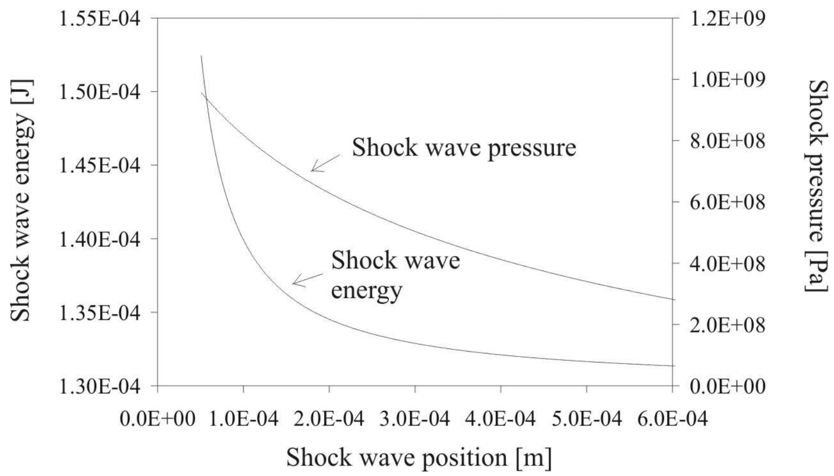

Fig. 3. Shock wave energy E and the shock wave pressure p as function of radial coordinate.

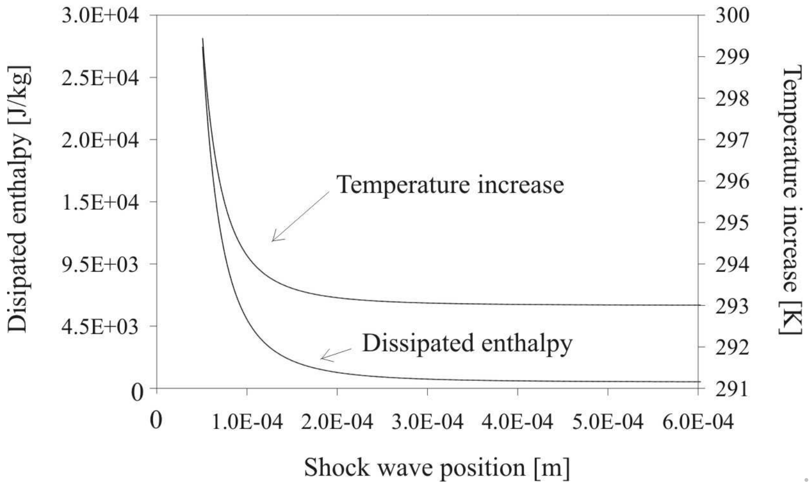

Fig.4. Dissipated enthalpy h_{dis} and the temperature increase T_1 as function of radial coordinate.

are presented in Fig. 3 and Fig. 4. From the figures is obvious that the pressure decrease is caused rather by the spherical divergence then by the dissipation effect as the shock wave energy decrease is only several percent from its initial value. The temperature of the liquid after passing the shock wave decreases proportionally to the shock peak pressure and the dissipated energy is maximal at the maximum pressure.

7. Conclusion

The paper presented a possibility of simulation of the propagation of spherical shock wave including the energy dissipation. For the solution, the dissipated enthalpy based on the new material data derived from NIST material database was derived. The presented model for the energy dissipation can be used for any liquids when the coefficients for Eqs. (2) – (5) are known. The comparison of the dissipated energy calculated by Richardson in [5] and the presented data shows very good agreement.

Acknowledgement

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