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Growing and remodeling material as a dynamical system

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Abstract

Contribution contains the short description of the general theory of growth and remodeling based on Di'Carlo's approach. This theory is applied to the one dimensional continuum using the quadratic form of the free energy function. Two different forms of loading are dealt – isometric and isotonic one. For both cases the corresponding equations describe the dynamical system. Its properties are analyzed using the methods of nonlinear dynamics. It's shown the influence of the constant growth and remodeling parameters on the stability of the equilibrium point.

Keywords: growth, remodeling, 1D continuum, isometric loading, isotonic loading, dynamical system, stability, equilibrium point

1. Introduction

Further we will assume growth as a volume change and remodeling as a change of material parameters or anisotropy. Then a lot of different changes in the material behavior can be thought as a growth or remodeling. As example we can take the growth of tissue during development and aging but also the changes in muscle during its stimulation. In the last case we can observe also some changes in its stiffness – kind of remodeling. Another example is e.g. the piezoelectric material – under influence of electric potential it changes its length.

In literature we can find different mathematical models of growth and remodeling. This contribution is based on the DiCarlo's theory [1]. This theory leads to the system of evolution differential equations for parameters describing volume, deformation or material properties. The form of these equations depends on the particular material. This is expressed by a set of material parameter (some of these are constants, some are time dependent). In any cases this system can be thought as a dynamical system and be analyzed with the adequate means. The properties of this system, e.g. the stability, depend on the above parameters. To find the critical values of these parameters is crucial from two points of view: At first same instabilities can occur in the modeled system and we can see, where is the source of this behavior. This can happen e.g. in the modeling of muscles using the mentioned theory. The question is if the causes of the observed instability of muscle under some outer conditions are the mechanical properties of the muscle tissue or the nerve control. The second reason is that when we identify the parameters using some results of experiment we need to know their limits.

This is the goal of this contribution. To be simple as possible the 1D continuum is taken in account. In [1] and [3] can be found some concrete application of the used growth theory and therefore here is introduced only very brief introduction and summarization of this theory. Main attempt is devoted to the analysis of some chosen types of dynamical systems.

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